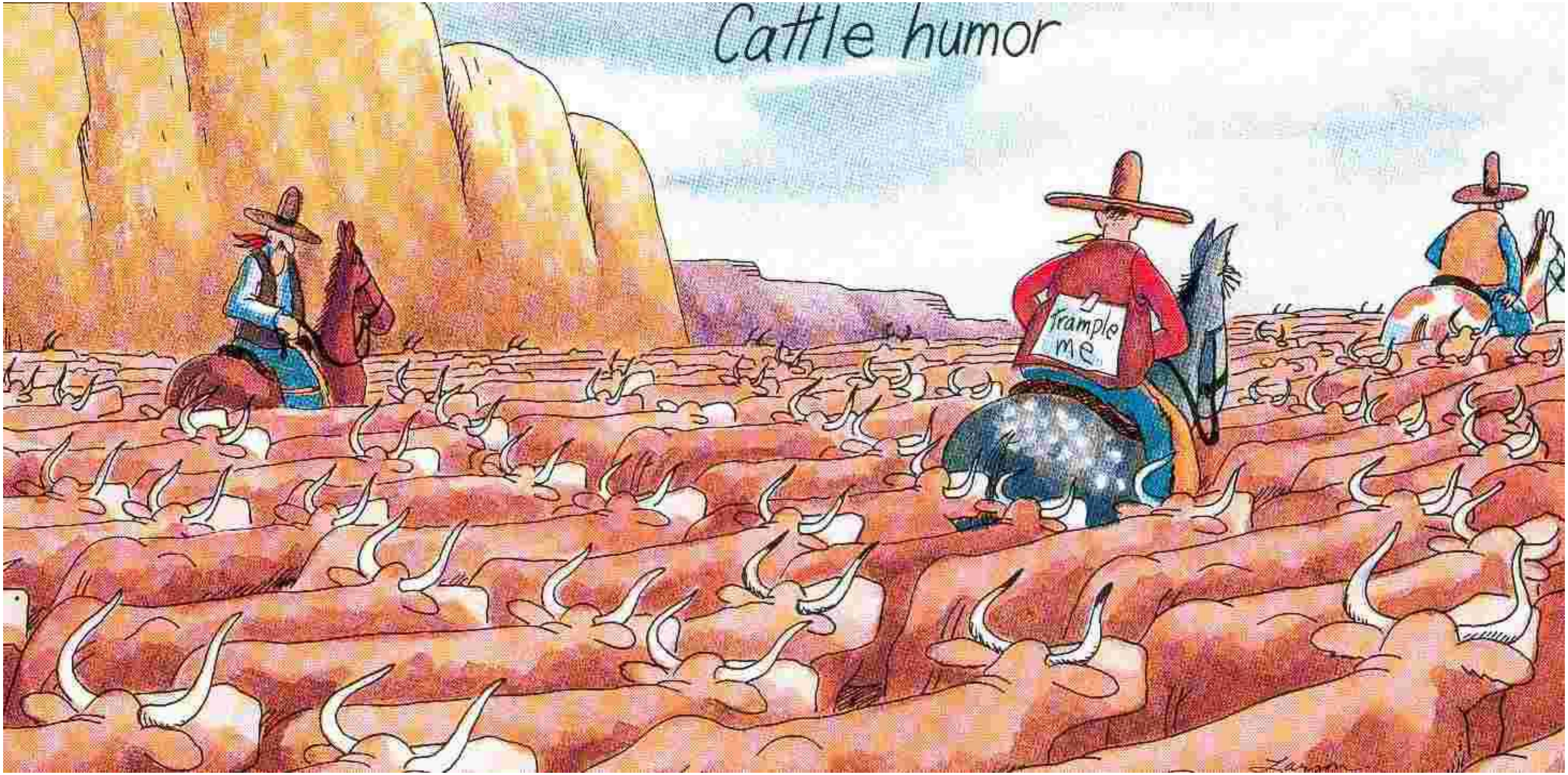
Analyse de données polytomiques



Introduction

Variables polytomiques

- Ordonnées (ordinales)
- Non ordonnées (nominales)

Variables ordinales

- Wright 1934 Polydactylie chez le cobaye
- Pearson 1990 Corrélation tétrachorique
- Article de base: P McCullagh 1980
  - Regression models for ordinal data, JRSS B, 42, 109-142
- Synthèse de Liu & Agresti, 2005

Exemple I/Données

Table1 : Calving scores in Blonde d'Aquitaine breed by gender of calf and parity of cow

Gender	Parity	I	II	III	Total
Male	1	55	65	33	153
	2	563	535	162	1260
	3	1990	1525	424	3939
	4	488	423	118	1029
	Total	3096 (48.5%)	2548 (39.9%)	737 (11.6%)	6381
Female	1	69	51	11	131
	2	761	433	61	1255
	3	2559	1216	141	3916
	4	300	178	22	500
	Total	3689 (63.6%)	1878 (32.4%)	235 (4.0%)	5802

1=First calving; 2=2nd calving, 3=3rd calving and beyond 4=unknown parity

Exemple 2/Données

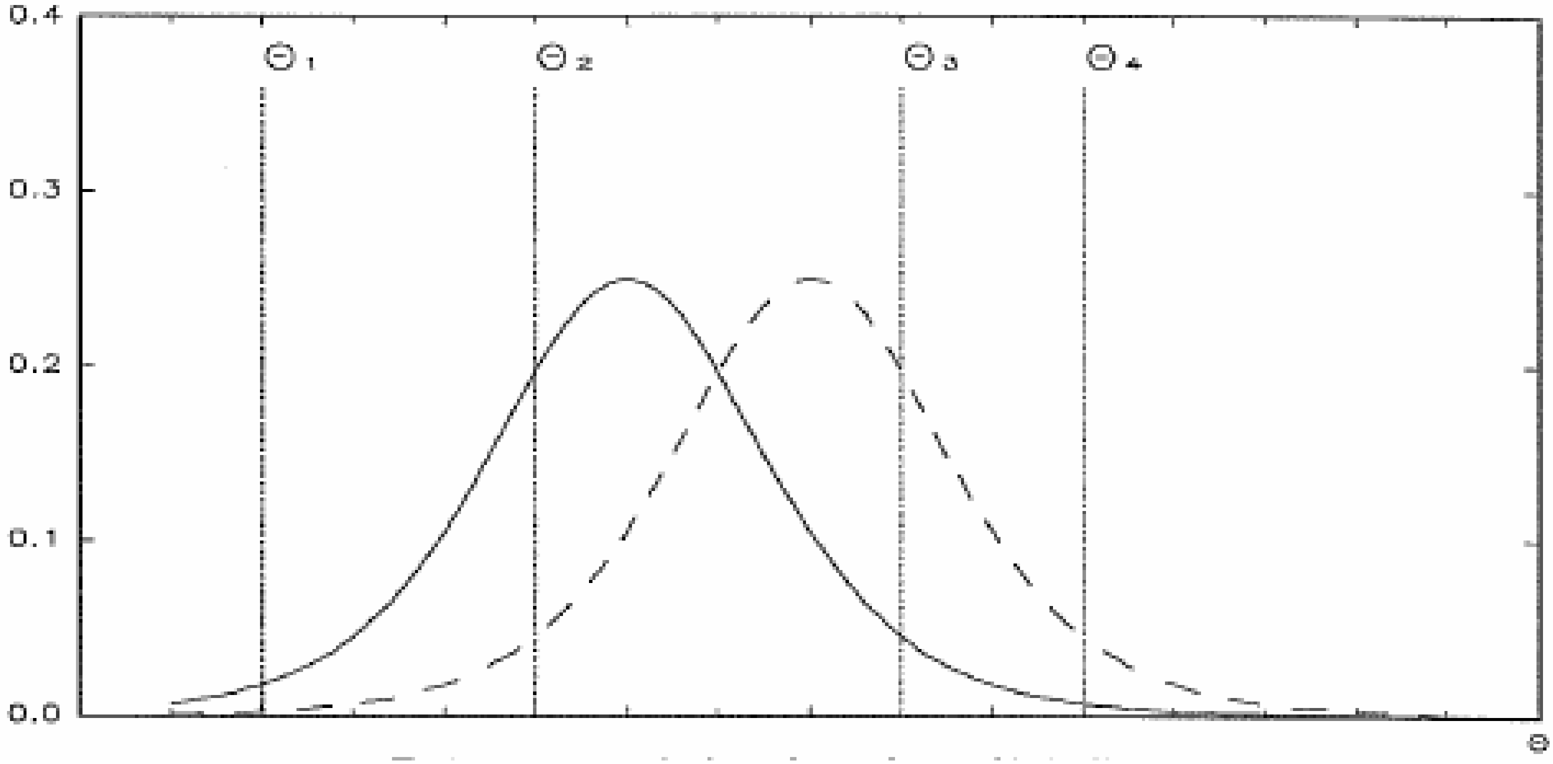
Sex	Parity	Sire	Score1	Score2	Score3
0	1	1	2	3	1
0	1	2	6	5	2
0	1	3	1	2	4
0	1	4	2	2	0
0	1	5	1	4	4
0	1	6	1	1	0
0	1	7	5	3	0
0	1	8	2	4	1
0	1	9	4	3	0
0	0	1	8	9	3
0	0	2	10	8	1
0	0	3	10	13	6
0	0	4	4	9	6
0	0	5	10	11	7
0	0	6	3	9	2
0	0	7	14	12	2
0	0	8	12	10	3
0	0	9	19	10	1
1	1	1	1	2	0
1	1	2	6	1	1
1	1	3	5	4	0
1	1	4	2	1	3
1	1	5	10	3	1
1	1	6	5	3	0
1	1	7	4	4	0
1	1	8	5	0	0
1	1	9	7	5	1
1	0	1	14	10	3
1	0	2	30	12	1
1	0	3	10	3	3
1	0	4	13	3	0
1	0	5	9	10	3
1	0	6	7	6	1
1	0	7	30	6	0
1	0	8	22	6	1
1	0	9	24	10	0

Sex: 0=Male, 1=Female; Parity: 1=Heifers, 0=Adult Cows

Catégories ordonnées/notations

Catégorie:	1	j	$J-1$	J
Strate:				
i	π_{i1}	π_{ij}	π_{iJ-1}	π_{iJ}
	$\kappa_{i1} = \pi_{i1}$	$\kappa_{ij} = \pi_{i1} + \ldots + \pi_{ij}$	$\kappa_{iJ-1} = \pi_{i1} + \ldots.\pi_{iJ-1}$	$\kappa_{iJ} = 1$

Modèle à seuils



Modèle à seuils

l_{ir} : observation r de la strate i

On postule l'existence d'une varaible latente $L_{ir} \sim \mathcal{N}\left(\eta_i, \sigma_i^2\right)$

munie de seuils: $\tau_1, \tau_2, ..., \tau_j, ..., \tau_{J\text{-}1}$

$$\kappa_{i1} = \pi_{i1} = \Pr(L_{ir} \leq \tau_1) = \Pr(\underbrace{\frac{L_{ir} - \eta_i}{\sigma_i}}_{\mathcal{N}(0,1)} \leq \frac{\tau_1 - \eta_i}{\sigma_i}) = \Phi\left(\frac{\tau_1 - \eta_i}{\sigma_i}\right)$$

$$\kappa_{i2} = \pi_{i1} + \pi_{i2} = \Pr(L_{ir} \leq \tau_2) = \Phi\left(\frac{\tau_2 - \eta_i}{\sigma_i}\right)$$

$$\sigma_i = \sigma = 1, \quad \boxed{\kappa_{ij} = \Phi\left(\tau_j - \eta_i\right)} \Leftrightarrow \tau_j - \eta_i = \Phi^{-1}\left(\kappa_{ij}\right)$$

Modèle à seuils

$$\boxed{\kappa_{ij} = \Phi\left(\tau_j - \eta_i\right)} \Leftrightarrow \tau_j - \eta_i = \Phi^{-1}\left(\kappa_{ij}\right)$$

$$\tau_j - \eta_i = \underbrace{\left(\tau_j - \alpha_R\right)}_{\substack{J-1 \text{ intercepts} \\ \text{ou "cut points"}}} - \underbrace{\mathbf{x}_i^{'}\boldsymbol{\beta}}_{\text{covar sans intercept}} - \mathbf{z}_i^{'}\mathbf{u}_i$$

$$\tau_j - \eta_i = \underbrace{\left(\tau_j - \tau_1\right)}_{\text{(J-2) seuils en écart au 1er}} - \underbrace{\left(\alpha_R - \tau_1\right)}_{\text{population de référence}} - \mathbf{x}_i^{'}\boldsymbol{\beta} - \mathbf{z}_i^{'}\mathbf{u}_i$$

Autres fonctions de liens

Plus généralement $\kappa_{ij} = \underbrace{F}_{CDF}\left(\tau_j - \eta_i\right) \Leftrightarrow \tau_j - \eta_i = F^{-1}\left(\kappa_{ij}\right)$

$F^{-1}(x)$:

1. logit : $\ln\left[x/(1-x)\right]$

2. probit : $\Phi^{-1}(x)$,

3. inverse de la Student (Cauchy)

1, 2 & 3 : "invariance palindromique" (inversion de l'ordre des catégories)

3. gompit : $\log(-\log(1-x))$ "complementary log log"

pas d'invariance palindromique "group Cox model"

Catégories ordonnées/Inférence

- Méthodes approchées
  - PQL, MQL
  - Proc SAS Glimmix
- Maximum de vraisemblance
  - Pas d'intégration (mod fixe)
    - Proc SAS-logistic
  - Quadrature de Gauss
    - Proc SAS-nlmixed (modèle fixe ou mixte)
  - EM stochastique
    - Natarajan et al, 2000, CSDA
- Approche bayésienne

Modèle à seuils probit normal/ML

```
proc logistic data=bascores descending ;
      title calving score gender parity gender*parity
fixed model ML via probit;
      freq count;
      class gender parity;
      model  score=gender parity gender*parity/
link=probit scale=none;
run;
```

Modèle à seuils probit normal/ML

Type 3 Analysis of Effects			
Effect	DF	Wald Chi-Square	Pr > ChiSq
gender	1	13.1933	0.0003
parity	3	31.3079	<.0001
gender*parity	3	1.3122	0.7262

Analysis of Maximum Likelihood Estimates							
Parameter			DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	3		1	-0.8735	0.0908	92.4952	<.0001
Intercept	2		1	0.4235	0.0906	21.8697	<.0001
gender	F		1	-0.4943	0.1361	13.1933	0.0003
parity	2		1	-0.2801	0.0958	8.5463	0.0035
parity	3		1	-0.4164	0.0922	20.4129	<.0001
parity	4		1	-0.3497	0.0971	12.9761	0.0003
gender*parity	F	2	1	0.0637	0.1439	0.1960	0.6579
gender*parity	F	3	1	0.0754	0.1387	0.2957	0.5866
gender*parity	F	4	1	0.1387	0.1506	0.8476	0.3572

Modèle à seuils probit normal/ML/SAS-Nlmixed

```
proc nlmixed data=bascores;
parms b0=0 b1=-0.5 b2=-0.5 b3=-0.5 b4=-0.5 b5=0 b6=0
b7=0 tau2=1;
bounds tau2>0;
theta=b0+b1*sex+b2*par2+b3*par3+b4*par4+b5*sex*par2+b
6*sex*par3+b7*sex*par4;
if (score=1) then z=probnorm(-theta);
else if (score=2) then
z=probnorm(tau2-theta)-probnorm(-theta);
else if (score=3) then
z=1-probnorm(tau2-theta);
if (z>1e-10) then ll=log(z);
else ll=-1e100;
model score~general(ll);
estimate 'inter1' -b0;
estimate 'inter2' tau2-b0;
replicate count;
run;
```

Modèle à seuils probit normal/ML/SAS-NImixed

Parameter	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper	Gradient
b0	0.4235	0.09087	24	4.66	<.0001	0.05	0.2359	0.6110	0.000213
b1	-0.4943	0.1363	24	-3.63	0.0013	0.05	-0.7755	-0.2130	0.000054
b2	-0.2801	0.09610	24	-2.91	0.0076	0.05	-0.4784	-0.08171	0.000121
b3	-0.4163	0.09246	24	-4.50	0.0001	0.05	-0.6072	-0.2255	0.000074
b4	-0.3497	0.09739	24	-3.59	0.0015	0.05	-0.5507	-0.1487	0.000032
b5	0.06369	0.1440	24	0.44	0.6623	0.05	-0.2336	0.3610	0.000063
b6	0.07542	0.1389	24	0.54	0.5921	0.05	-0.2112	0.3621	0.000017
b7	0.1387	0.1508	24	0.92	0.3668	0.05	-0.1725	0.4498	-4.01E-6
tau2	1.2969	0.01719	24	75.47	<.0001	0.05	1.2615	1.3324	-0.00015

Additional Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper
inter1	-0.4235	0.09087	24	-4.66	<.0001	0.05	-0.6110	-0.2359
inter2	0.8735	0.09113	24	9.59	<.0001	0.05	0.6854	1.0615

Catégories ordonnées/Bayes/Réf.

- MAP & approximations
  - GFCAT: Gianola Foulley, 1983, GSE
  - Stiratelli et al, 1984 (binary), Biometrics
- Posterior inference
  - Albert & Chib, 1993, JASA
  - McCulloch & Rossi, 1994, J Econometrics
  - Sorensen et al, 1995, GSE
  - Chipman & Hamada, 1996, Technometrics
  - Girard & Parent, 2000, Technometrics
  - Sorensen & Gianola, 2002

Modèle à seuils probit normal/Bayes

Soit $\phi = (\boldsymbol{\beta}', \mathbf{u}')'$ loi a priori $[\mathbf{L}, \boldsymbol{\tau}, \phi, \mathbf{G},]$

Lois a priori $[\mathbf{L}, \phi, \mathbf{G}, \boldsymbol{\tau}] = [\mathbf{L}, \phi \mid \mathbf{G}, \boldsymbol{\tau}][\mathbf{G}][\boldsymbol{\tau}]$

$[\mathbf{L} \mid \phi, \mathbf{G}] \sim$ Normales indtes

1er niveau hiérarchie: modèle var latente

$[\boldsymbol{\tau}] \sim$ Uniforme (ou normales) pour stat d'ordre

$[\phi \mid \mathbf{G}] \sim$ Multinormales

2ème niveau hiérarchie: effets fixes et aléatoires

$[\mathbf{G}^{-1}] \sim$ Wishart (gamma) ou Jeffreys

3ème niveau hiérarchie: hyperparamètres

Modèle à seuils probit normal/Bayes

Soit $\phi = (\boldsymbol{\beta}', \mathbf{u}')'$ loi a posteriori $[\mathbf{L}, \phi, \mathbf{G}, \boldsymbol{\tau} \mid \mathbf{y}]$

Lois conditionnelles

$[\mathbf{L} \mid \phi, \mathbf{y}] \sim$ Normales tronquées indtes entre $(\tau_{j\text{-}1}, \tau_j)$

$[\boldsymbol{\tau} \mid \phi, \mathbf{L}, \mathbf{y}] \sim$ Uniforme tronquée

ex $\tau_j \sim \mathrm{U}(\max \mathrm{L} \mid y_i = j, \min \mathrm{L} \mid y_i = j+1)$

$[\phi \mid \mathbf{G}, \mathbf{L}] \sim$ Multinormales (théorie BLUP)

$[\mathbf{G}^{-1} \mid \mathbf{L}, \phi] \sim$ Wishart (ou gamma)

Modèle probit cumulé/estimations

Table 2: Parameters estimates under a standard TM (probit)

		ML	Posterior Mean
Intercept	1	-0.423±0.091	-0.420±0.089
	2	0.874±0.091	0.877±0.089
Gender	F vs M	-0.494±0.136	-0.496±0.129
Parity	2 vs 1	-0.280±0.096	-0.277±0.095
	3 vs 1	-0.416±0.092	-0.413±0.091
	4 vs 1	-0.350±0.097	-0.349±0.097
GxP	F2	0.064±0.144	0.065±0.137
	F3	0.075±0.139	0.077±0.132
	F4	0.139±0.151	0.143±0.146
X2		21.9 (7)	
P-value		0.026	0.024*

ML via SAS Proc-Logistic, Posterior mean via Winbugs non informative
*PPP-value

- « Inference under a particular model should be Bayesian »

- « Model assessment can and should involve frequentist ideas »

R Little (2006) Calibrated Bayes: A Bayes/frequentist Roadmap (2005 ASA presidential address), TAS, 60, 213-223

Posterior Predictive Check

-Confrontation des données observées $\mathbf{y}$ et de réplications $\mathbf{y}^{rep}$

-Les réplications sont obtenues par tirage dans la distribution prédictive a posteriori sachant le modèle

$$p(\mathbf{y}^{rep} \mid \mathbf{y}) = \int p(\mathbf{y}^{rep}, \boldsymbol{\theta} \mid \mathbf{y})\, d\boldsymbol{\theta} = \int p(\mathbf{y}^{rep} \mid \boldsymbol{\theta}, \not{\mathbf{y}})\, \pi(\boldsymbol{\theta} \mid \mathbf{y})\, d\boldsymbol{\theta}$$

-Elles intégrent l'incertitude liée à l'estimation des paramètres $\boldsymbol{\theta}$

Posterior Predictive Check/suite

-Critère de discrépance $\mathrm{T}(\mathbf{y},\boldsymbol{\theta})$

Gelman, Meng & Stern, 1996; Stern, 2000,

Gelman, Carlin, Stern, Rubin, 2004, Chap6

-Confronter la distr. de $\mathrm{T}(\mathbf{y}^{rep},\boldsymbol{\theta})\,|\,\mathbf{y}$ à celle de $\mathrm{T}(\mathbf{y},\boldsymbol{\theta})\,|\,\mathbf{y}$

$$\boxed{\text{PPP-value} = \Pr\left[\mathrm{T}(\mathbf{y}^{rep},\boldsymbol{\theta}) > \mathrm{T}(\mathbf{y},\boldsymbol{\theta})\,|\,\mathbf{y}\right]}$$

$$\text{Classical } P_C\text{-value} = \Pr\left[\mathrm{T}(\mathbf{y}^{rep}) > \mathrm{T}(\mathbf{y})\,|\,\boldsymbol{\theta},\mathrm{H}\right]$$

$\boldsymbol{\theta}$: null value or point estimate

PPP-value

$$\boxed{\text{PPP-value} = \Pr\left[\mathrm{T}\left(\mathbf{y}^{rep},\boldsymbol{\theta}\right) > \mathrm{T}\left(\mathbf{y},\boldsymbol{\theta}\right) \mid \mathbf{y}\right]}$$

$$\text{PPP-value} = \int\int \mathrm{I}_{\left[\mathrm{T}\left(\mathbf{y}^{rep},\boldsymbol{\theta}\right) > \mathrm{T}\left(\mathbf{y},\boldsymbol{\theta}\right)\right]}\, p(\mathbf{y}^{rep} \mid \boldsymbol{\theta})\, p(\boldsymbol{\theta} \mid \mathbf{y})\, d\mathbf{y}^{rep}\, d\boldsymbol{\theta}$$

$$\text{PPP-value} = \mathrm{E}\left[\mathrm{I}_{\left[\mathrm{T}\left(\mathbf{y}^{rep},\boldsymbol{\theta}\right) > \mathrm{T}\left(\mathbf{y},\boldsymbol{\theta}\right)\right]} \mid \mathbf{y}\right]$$

M tirages $\boldsymbol{\theta}^{m}$ $(m = 1,..,M)$ dans $\left[\boldsymbol{\theta} \mid \mathbf{y}\right]$ donc dans la prédictive $\left[\mathbf{y}^{rep} \mid \mathbf{y}\right]$

PPP-value=proportion des tirages pour lesquels $\mathrm{T}\left(\mathbf{y}^{rep,m},\boldsymbol{\theta}^{m}\right) > \mathrm{T}\left(\mathbf{y},\boldsymbol{\theta}^{m}\right)$

PPP/suite

- « Extreme probabilities clearly indicate that data are inconsistent with the model » Stern, 2000
- « Finding an extreme p value & thus rejecting the model is never an end of an analysis; the departure of the test quantity in question from its posterior predictive distribution will often suggests improvements of the model or places to check the data » Gelman et al, 2004

PPP-value/critère de Pearson

Ici : 1ère étape de la hiérarchie étant multinomiale, on peut prendre le critère de Pearson:

$$\mathrm{T}\left(\mathbf{y},\boldsymbol{\theta}\right)=\sum\nolimits_{ik}\left(O_{ik}-E_{ik}\right)^{2}/E_{ik}$$

PPP/Code Winbugs

```
for (i in 1:I) {
  YREP[i, 1:K]~dmulti(p[i,1:K],n[i])
for (k in 1:K) {
        E[i, k]<-n[i]*p[i,k]
        TREP[i, k]<-pow(YREP[i, k]-E[i, k],2)/E[i, k]
        TOBS[i, k]<-pow(Y[i, k]-E[i, k],2)/E[i, k]
        T[i, k]<-TREP[i, k]-TOBS[i, k]
                } }
        DT<-sum(T[,])
        PPP<-step(DT)
```

Modèle probit cumulé/qualité d'ajustement

Table 3: Goodness of fit of threshold models applied to calving scores

Observed (O) vs Expected (E)						$X=(O-E)/\sqrt{E}$		
I		II		III		I	II	III
55	51.7	65	72.1	33	29.3	-0.48	+0.83	-0.75
563	558.2	535	545.0	162	156.8	-0.22	+0.43	-0.43
1990	1958.0	1525	1592.0	424	388.4	-0.72	+1.69	-1.83
488	484.3	423	430.7	118	113.9	-0.18	+0.37	-0.40
69	69.3	51	50.4	11	11.3	+0.02	-0.10	+0.03
761	769.3	433	414.6	61	71.1	+0.29	-0.91	+1.18
2559	2583.0	1216	1161.0	141	171.5	+0.48	-1.62	+2.31
300	305.5	178	165.8	22	28.71	+0.30	-0.96	+1.23

Modèle à seuils à variance hétérogène (HTM)

McCullach, 1982, JRSS;Foulley & Gianola, 1996, GSE;

Jaffrézic Robert Foulley, 1999, GSE; Derquenne, 1995, 2007

Soit π_{ij} probabilité de réponse dans la categorie j pour une observation r de la strate i

$$\pi_{ij} = \Phi\left(\frac{\tau_j - \eta_i}{\sigma_i}\right) - \Phi\left(\frac{\tau_{j-1} - \eta_i}{\sigma_i}\right)$$

$\eta_i = \mathbf{x}_i'\boldsymbol{\beta}$ or $\eta_i = \mathbf{x}_i'\boldsymbol{\beta} + \mathbf{z}_i'\mathbf{u}$; $\ln(\sigma_i) = \mathbf{p}_i'\boldsymbol{\delta}$

Equivaut à:

$l_{ir} = \mathbf{x}_i'\boldsymbol{\beta} + \mathbf{z}_i'\mathbf{u} + \sigma_i e_{ir}^*$ and $y_{ijr} = 1 \Leftrightarrow \tau_{j-1} < l_{ir} \leq \tau_j$

$\Phi(x)$ peut etre remplacé par $F(\mathrm{x}) = (1 + \exp(-x))^{-1}$

Modèle à seuils hétérogène/estimations

Table 6: Parameters estimates under a heteroskedastic TM (probit)

		ML	Posterior Mean
Intercept	1	-0.350±0.065	-0.348±0.064
	2	0.889±0.065	0.893±0.068
Gender	F vs M	-0.341±0.026	-0.342±0.026
Parity	2 vs 1	-0.236±0.070	-0.234±0.066
	3 vs 1	-0.359±0.065	-0.357±0.064
	4 vs 1	-0.274±0.070	-0.273±0.069
Dispersion	F vs M	-0.123±0.027	-0.121±0.027
X2		3.65 (9)	
P-value		0.93	0.79*
DIC			122 (7) vs 144 (9)

ML via personal APL programme, Posterior mean via Winbugs non informative

*PPP-value

Modèle à seuils hétérogène/ajustement

Table 5: Goodness of fit of threshold models applied to calving scores

Observed (O) vs Expected (E)						$X=(O-E)/\sqrt{E}$		
I		II		III		I	II	III
55	55.6	65	68.8	33	28.7	0.06	+0.45	-0.84
563	572.6	535	523.4	162	164.0	0.39	-0.51	+0.14
1990	1983.0	1525	1539.0	424	417.2	-0.16	+0.35	-0.35
488	483.6	423	419.7	118	125.7	-0.21	-0.16	+0.67
69	65.0	51	55.1	11	10.8	-0.51	+0.55	-0.08
761	754.8	433	438.8	61	61.4	-0.23	+0.27	+0.02
2559	2561.0	1216	1213.0	141	142.4	+0.03	-0.07	+0.09
300	309.2	178	168.5	22	22.3	+0.52	-0.74	+0.05

Exemple 2/Données

Sex	Parity	Sire	Score1	Score2	Score3
0	1	1	2	3	1
0	1	2	6	5	2
0	1	3	1	2	4
0	1	4	2	2	0
0	1	5	1	4	4
0	1	6	1	1	0
0	1	7	5	3	0
0	1	8	2	4	1
0	1	9	4	3	0
0	0	1	8	9	3
0	0	2	10	8	1
0	0	3	10	13	6
0	0	4	4	9	6
0	0	5	10	11	7
0	0	6	3	9	2
0	0	7	14	12	2
0	0	8	12	10	3
0	0	9	19	10	1
1	1	1	1	2	0
1	1	2	6	1	1
1	1	3	5	4	0
1	1	4	2	1	3
1	1	5	10	3	1
1	1	6	5	3	0
1	1	7	4	4	0
1	1	8	5	0	0
1	1	9	7	5	1
1	0	1	14	10	3
1	0	2	30	12	1
1	0	3	10	3	3
1	0	4	13	3	0
1	0	5	9	10	3
1	0	6	7	6	1
1	0	7	30	6	0
1	0	8	22	6	1
1	0	9	24	10	0

Sex: 0=Male, 1=Female; Parity: 1=Heifers, 0=Adult Cows

Modèle à seuils mixte homogène/estimations

Table 5: Estimates of fixed effects and prediction of sire effects

	PQL approach		Posterior inference	
	Estimate	SE	Estimate*	SE*
Inter1	-0.198	0.121	-0.199	0.130
Inter2	1.051	0.128	1.053	0.137
Gender: F-M	-0.586	0.114	-0.587	0.114
Parity: H-C	0.162	0.163	0.162	0.164
GxP: FC	-0.026	0.231	-0.026	0.233
Sire 1	0.153	0.154	0.147	0.160
Sire 2	-0.175	0.145	-0.171	0.153
Sire 3	0.217	0.150	0.209	0.161
Sire 4	0.168	0.161	0.162	0.170
Sire 5	0.274	0.144	0.265	0.157
Sire 6	0.125	0.168	0.120	0.173
Sire 7	-0.312	0.147	-0.305	0.164
Sire 8	-0.181	0.151	-0.178	0.161
Sire 9	-0.269	0.145	-0.263	0.159
Sire Variance**	0.0699	0.0455	0.070	0.082
Model check***			0.369	
DIC (pD)			245 (11.5)	

mean and standard deviation of the posterior distribution; ** exp(MAP log(sire var))
***PPP-value

Modèle à seuils mixte hétérogène/estimations

Table: Estimates of fixed effects and prediction of sire effects under a HTM (sex effect)

	Posterior inference	
	Estimate*	SE*
Inter1	-0.218	0.137
Inter2	1.094	0.147
Gender	-0.680	0.158
Parity	0.167	0.165
GxP	-0.015	0.251
Disp/ Gender F-M	0.117	0.123
Sire 1	0.150	0.173
Sire 2	-0.189	0.168
Sire 3	0.230	0.172
Sire 4	0.186	0.183
Sire 5	0.292	0.168
Sire 6	0.138	0.188
Sire 7	-0.323	0.173
Sire 8	-0.179	0.172
Sire 9	-0.297	0.170
Sire Variance	0.081#	0.087
Model check**	0.403	
DIC	246 (12)***	

mean and standard deviation of the posterior distribution;
PPP-value * 245 (11.5) pour le modèle à seuils homogène
#exp(MAP logvar)

Discussion

- Autres modèles
  - Modèles logit séquentiels
    - Tutz, 1990, 1991
  - Modèles logistiques PLS
    - Bastien et al, 2005
- Modèles multivariés
  - Plusieurs variables ordinales
  - Variables ordinales et continues
- Inférence
  - Approchées vs exactes
  - Modèles marginaux (GEE) vs modèles mixtes

Bibliographie

Fahrmeir L, Tutz G (1994) Multivariate statistical modelling based on GLM, Springer-Verlag

Liu I, Agresti A (2005) The analysis of ordered categorical data: an overview and a survey of recent developments. Societas de Estadistica e Investigacion Operativa, 14, 1-73

Marin JM, Robert CP (2007) A practical approach to computational Bayesian statistics, Springer Verlag

Molenberghs G, Verbeke G (2005) Models for discrete longitudinal data. Springer-Verlag

McCullagh P (1980) Regression models for ordinal data. JRSS B 42, 109-142

Stokes ME, Davis CS, Koch GG (2000) Categorical data analysis using the SAS system, 2nd edition, SAS Institute #57998