Combining cheap massive commercial data and unbiased scientific survey: a zero inflated model under preferential sampling

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Fisheries Management

Reliable stock assesment require reliable relative abundance indices :

- Unbiased or with constant multiplicative bias,
- As precise as possible (low variability)
- Acceptable for stakeholders

Major data sources:

- Scientific survey data,
- Commercial fisheries data.

Scientific Survey data

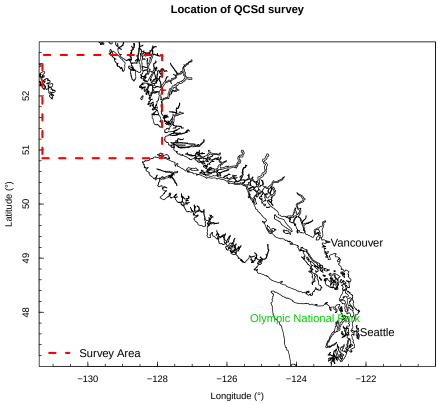


Figure: Queen Charlotte area - DFO (GroundFish division) - Focus on Dover Sole

Scientific Survey data

- Scientific campaigns are organized regularly to monitor the species of interest.
- Mostly random sampling or stratified random sampling design.
- Produce unbiased but highly variable and expensive abundance indices series.
- Stakeholders have difficulty to accept random sampling : "why sample some zone where there is no fish"?

Scientific Survey data

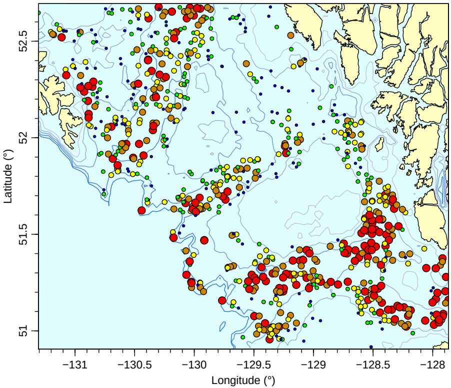


Figure: QCSd - The records are the weights of Dover Sole caught.

Commercial Fisheries data

- Cheap and massive data.
- Roughly used, produce biased abundance indices.
- Stakeholders are part of the collection process.

Commercial Fisheries data

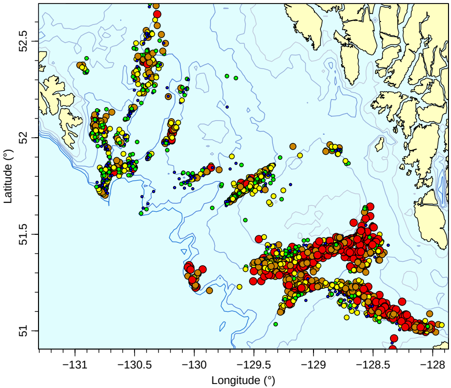
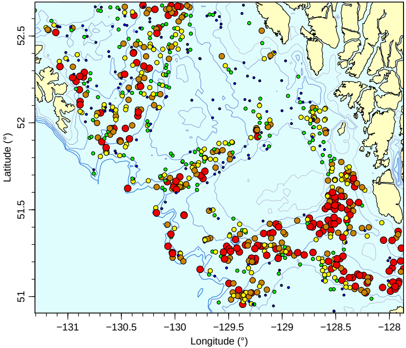
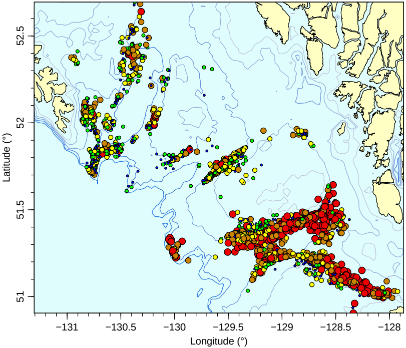


Figure: Com. Fish.: The records are the weight of Dover Sole Catch

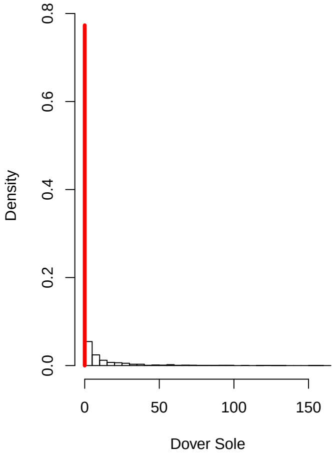
Two data sources





FISHING.ID	YEAR	LAT	LONG	SWEPT.AREA	DOVER.SOLE
1700	1999	51.43333	-129.2117	0.770	0.0
6550	2005	51.06167	-128.2867	0.610	210.1
...					

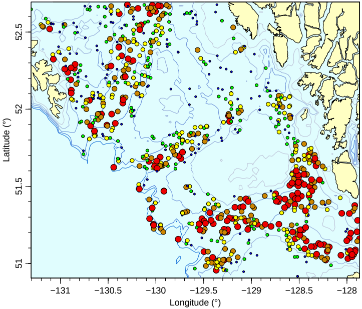
Zero-Inflated data



Continuous Zero Inflated data

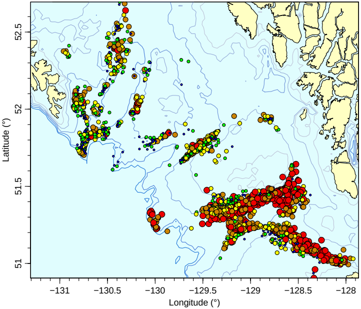
- Classically, high proportion of zeros,
- Appart from 0, continuous biomass data

Data are spatially correlated



- The biomass repartition is somehow continuous,
- Data are spatially correlated,
- This correlation should be accounted for.

Location of Commercial Fisheries catch



- Fishermen target specific species,
- Location of the catch are highly related to the amount of local biomass,
- This information must be accounted for.

Modelling challenges

The resulting model should represent,

- Zero inflated,
- Spatially correlated,
- and preferentially sampled,

data,
for building a relative abundance index.

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Taking benefits of hierarchical modelling

Biomass Model: Log Gaussian Cox Process

Let $\mu(s)$ be the local abundance and define the intensity of an inhomogenous Poisson Process which may be thought as the fish repartition.

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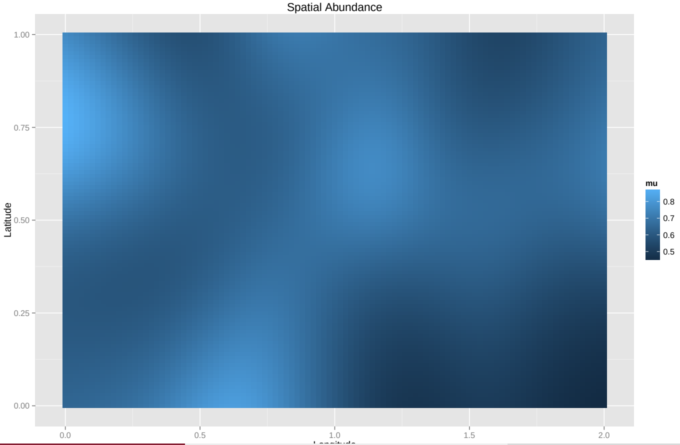

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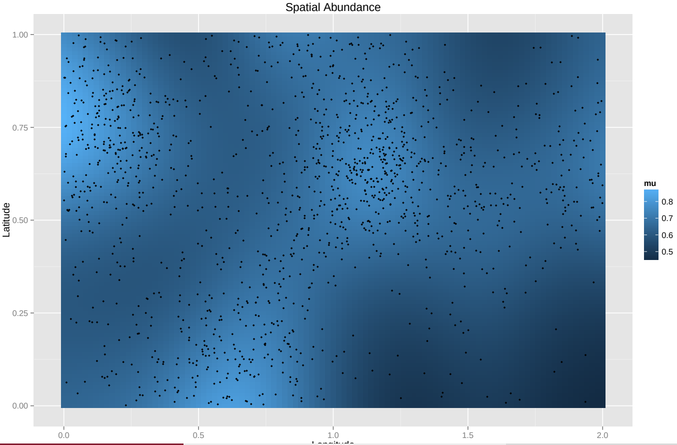

Biomass Model: Log Gaussian Cox Process

Let $\mu(s)$ be the local abundance and define the intensity of an inhomogenous Poisson Process which may be thought as the fish repartition.
To account for heterogeneity,

$$log(\mu(s)) = \alpha_0 + Z(s),$$

where $Z(s)$ is a gaussian random field (GRF) with covariance function $c(s,t) = \exp - \frac{d(s,t)^2}{2\phi^2}$

Observation Process : Compound Poisson Process

One fishing event, for a given swept area A :

$$N(A) \sim \mathcal{P}\left(\int_A \mu(s)ds\right),$$

is the number of fish caught.

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Approximation:

$$\int_A \mu(s)ds \approx |A|\mu_{s_A}.$$

$$Y(A) = \sum_{i=1}^{N(A)} \xi_i,$$

where ξ_i are iid random variable (weight).

Observation Process : Compound Poisson Process

Commercial

$$Y_s^C = \sum_{i=1}^{N_s^C} \xi_{s,i},$$
$$N_s^C \sim \text{Poisson}(||A_s|\mu_s),$$
$$\xi_{s,i}^C \sim \text{Exp}(\rho),$$

$$\mathbb{E}(Y_s^C) = \frac{\mu_s}{\rho}$$

Scientific

$$Y_s^S = \sum_{i=1}^{N_s^S} \xi_{s,i},$$
$$N_s^S \sim \text{Poisson}(|A_s|\mu_s),$$
$$\xi_{s,i}^S \sim \text{Exp}(\rho^S), \quad \rho^S = q\rho$$

$$\mathbb{E}(Y_s^S) = \frac{q\mu_s}{\rho}$$

$$\mathbb{P}(Y_s^i = 0) = \exp\left(-|A_s|\mu(s)\right)$$

called LOL model in Ancelet & al, 2010; Lecomte & al 2013

Full model specification

Process model :

$$log(\mu(s)) = \alpha_0 + Z(s),$$

where $Z(s)$ GRF with covariance function $c(s,t) = \exp -\frac{d(s,t)^2}{2\phi^2}$.

Data model :

$$Y_s^k \sim LOL(\mu_s, \rho^k)$$

But dimension issues when the number of observations increase.
Reduction dimension using random basis function.

A 2 D discrete convolution of a gridded (latent) structure

The points of the grid are denoted $g = 1..G$.

$$X(g) \underset{iid}{\sim} N(0, \sigma_x^2)$$

Convolution kernel K_θ between any data point s and grid location g

$$K_\theta(s, g) = \exp - \frac{d^2(s, g)}{\phi^2}$$

Discrete convolution for site $s, s = 1..S$:

$$Z(s) = \sum_{g=1}^{G} K_\theta(s, g) X(g) + m(s)$$

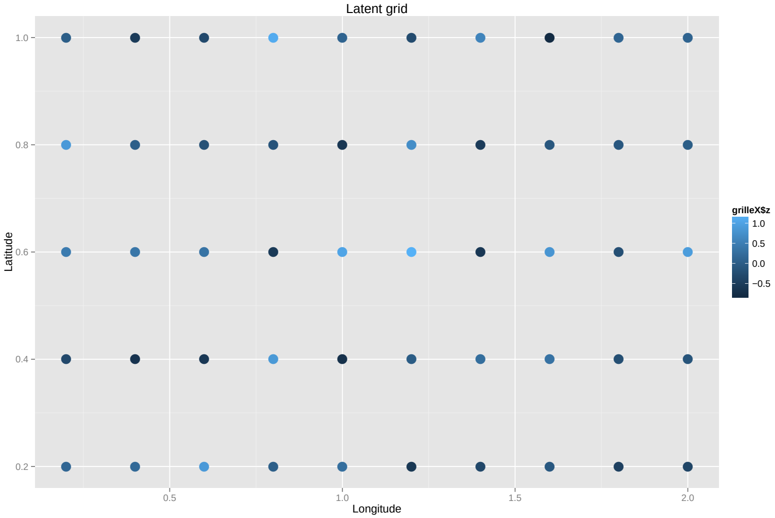
$$m(s) = \alpha_0 + \alpha_1 \times Depth(s) + \dots$$

Preferential sampling

Commercial fisheries focus on area with high abundance. The position of the commercial catch are modeled as an inhomogenous Poisson point process conditionned to have *NCom* points.

$$(S_1^C, \ldots, S_{NCom}^C) \sim IPP(\mu(s))$$

Full model specification with graphics



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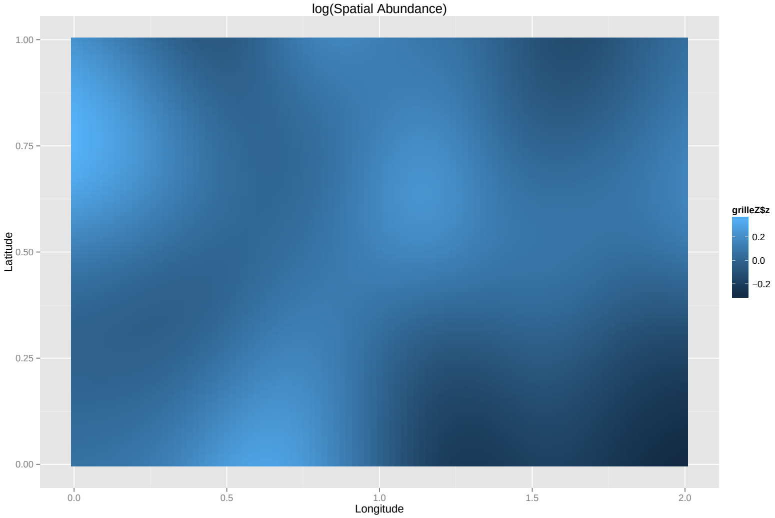

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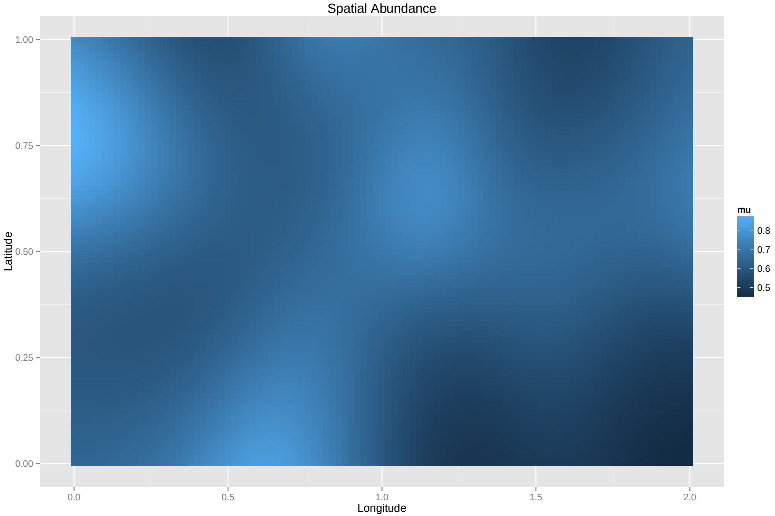

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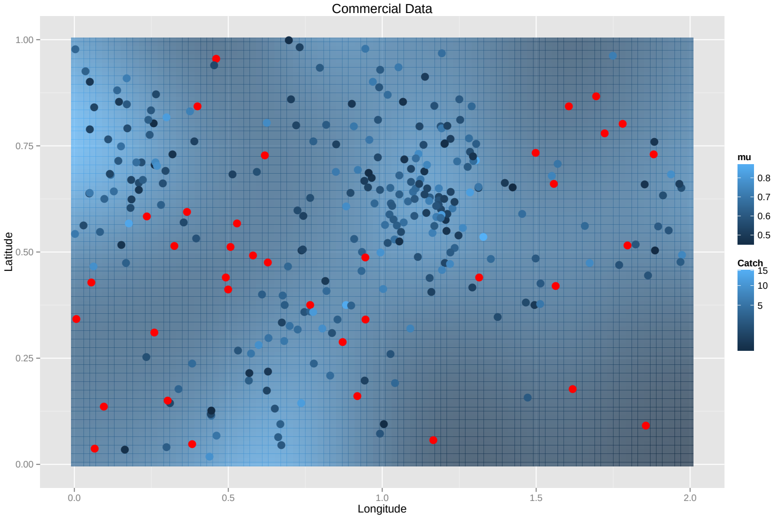

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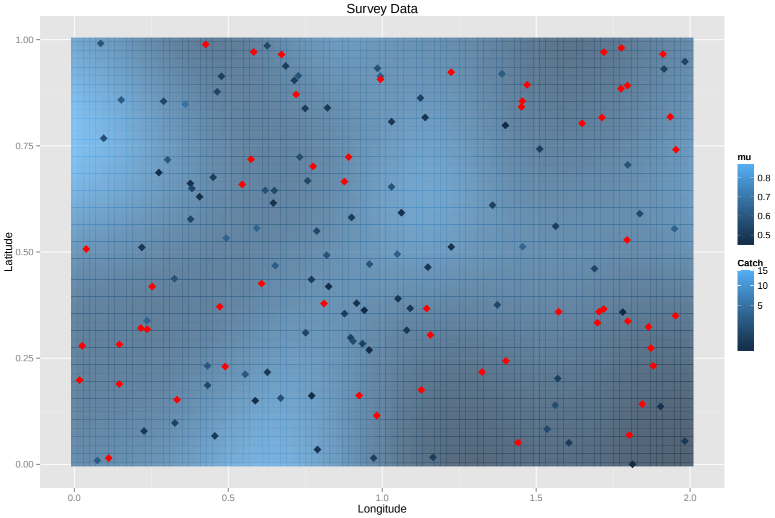

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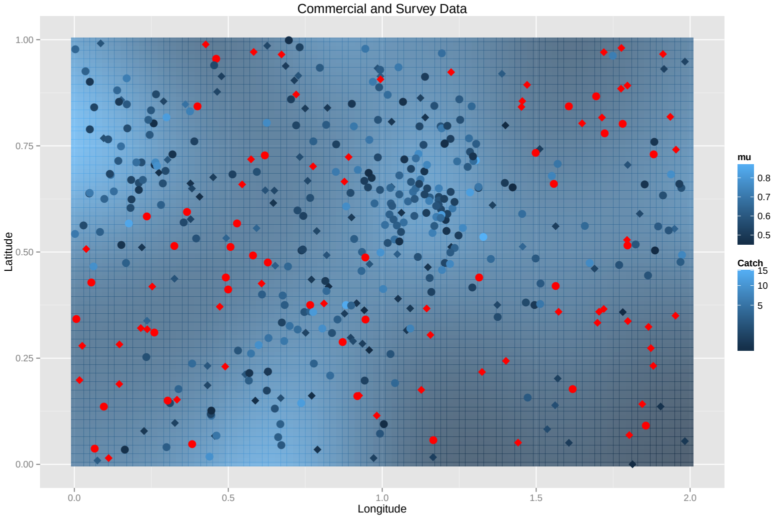

Model Summary

Data:
$\mathbf{S}^C = (S_1^C, \ldots, S_{nCom}^C)$, Commercial locations of catch : Poisson Process
$\mathbf{Y}^C = (Y_1^C, \ldots, Y_{nCom}^C)$, Actual commercial catch : LOL model
$\mathbf{Y}^S = (Y_1^S, \ldots, Y_{nScien}^S)$, Actual survey catch: LOL model

Latent layer:
$\mathbf{Z} = K_\phi \mathbf{X}$, with $\mathbf{X} = (X_1, \ldots, X_G)$ Independant centered gaussian variables, with variance σ^2.

Parameters:
$\theta = (\sigma^2, \phi, \boldsymbol{\alpha}, \rho^C, \rho^S)$

Indice:
$I = \int_s \mu(s) ds$

Problems - Likelihood

Complete likelihood

$$[\mathbf{Y}^C, \mathbf{Y}^S, \mathbf{S}^C, \mathbf{X}|\theta] = [\mathbf{Y}^C|\mathbf{S}^C, \mathbf{X}, \theta]\ [\mathbf{Y}^S|\mathbf{S}^S, \mathbf{X}, \theta]\ [\mathbf{S}^C|\mathbf{X}, \theta][\mathbf{X}|\theta]$$

Likelihood

$$[\mathbf{Y}^C, \mathbf{Y}^S, \mathbf{S}^C|\theta] = \int_{\mathbf{X}} [\mathbf{Y}^C, \mathbf{Y}^S, \mathbf{S}^C, \mathbf{X}|\theta] d\mathbf{X}$$

Computing the likelihood

Computing the likelihood

Monte Carlo approximation

$$MC : [\mathbf{Y}^C, \mathbf{Y}^S, \mathbf{S}^C|\theta] \approx \frac{1}{M}\sum_{m=1}^{M}[\mathbf{Y}^C, \mathbf{Y}^S, \mathbf{S}^C|\mathbf{X^m}, \theta], \quad \mathbf{X^m} \sim [X|\theta]$$

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Importance sampling approximation

$$IS : [\mathbf{Y}^C, \mathbf{Y}^S, \mathbf{S}^C|\theta] \approx \frac{1}{M}\sum_{m=1}^{M}\frac{[\mathbf{Y}^C, \mathbf{Y}^S, \mathbf{S}^C|\mathbf{X^m}, \theta][\mathbf{X^m}|\theta]}{q_\theta(\mathbf{X^m})}, \quad \mathbf{X^m} \sim q_\theta(.)$$

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  - propose $\theta^* \sim q(\cdot|\theta)$ and $X^* \underset{iid}{\sim} \prod_{m=1}^{M} q^S(Z^{(m)}|\theta^*)$
  - accept them with probability $\rho((\mathbf{X},\theta),(\mathbf{X}^*,\theta^*)) = \frac{\tilde{\pi}(\theta^*,\mathbf{Z}^*)}{\tilde{\pi}(\theta,\mathbf{Z})} \times \frac{q(\theta|\theta^*)}{q(\theta^*|\theta)}$

with $\tilde{\pi}(\theta,\mathbf{Z})$ given by the importance sampling quantity

$$\tilde{\pi}(\theta,\mathbf{Z}) = \frac{1}{M}\sum_{m=1}^{M} \frac{[\mathbf{Y}^C,\mathbf{Y}^S,\mathbf{S}^C,\mathbf{Z}^{(m)},\theta]}{q^S(\mathbf{Z}^{(m)}|\theta)}$$

Finding good importance function q_θ

Moment based method with kernel smoothing :

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$$\hat{Z}(s) = log\left(\frac{\rho\hat{Y}_s}{A_s}\right), \quad \hat{Y}(s) = K_{smooth}\mathbf{Y}$$

2. And $P(Y_s = 0) = \exp\{-|A_s|\mu(s)\}$,

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3. Finally, $Z = KX$,
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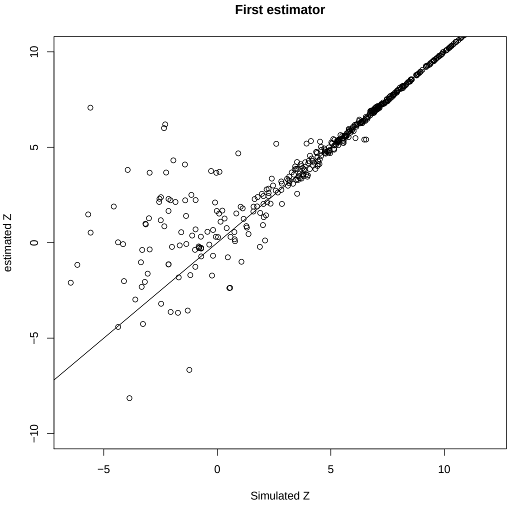
$$X^* \sim \mathcal{N}(\hat{X}, \Sigma_X)$$

Finding good importance function q_θ



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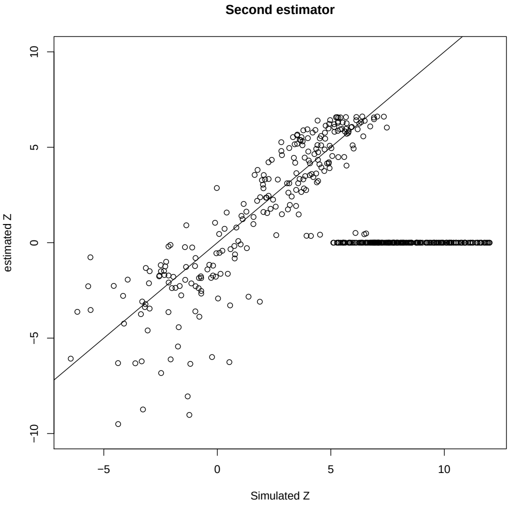

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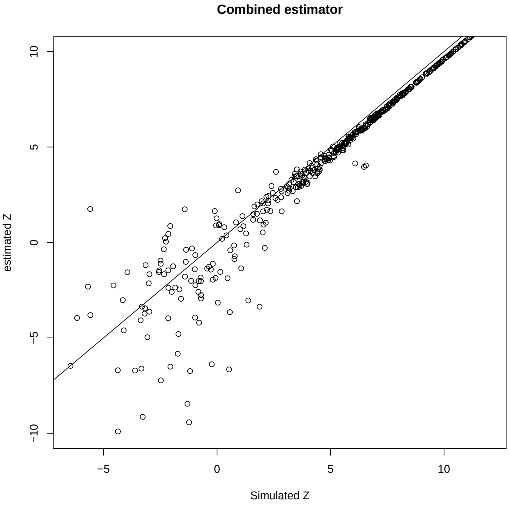

And now

Mixing all the ingredients and baking the cake

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