

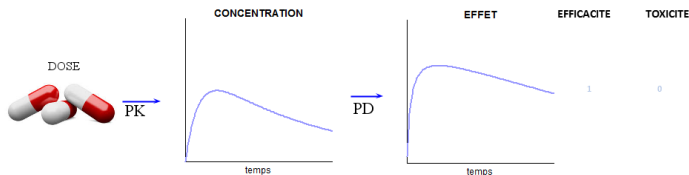
# Bayesian approaches for pharmacogenetic models with JAGS and Stan

Julie Bertrand, Maria De Iorio, David J. Balding



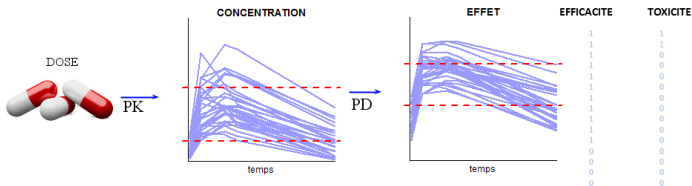
# Pharmacological and genetic variability

- Clinical pharmacology: study the interaction between the organism and the drug
  - pharmacokinetics (PK) and pharmacodynamics (PD)



# Pharmacological and genetic variability

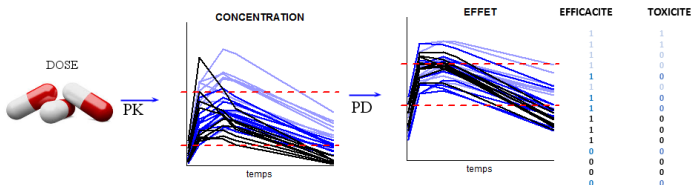
- Clinical pharmacology: study the interaction between the organism and the drug and its variability
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# Pharmacological and genetic variability

- Clinical pharmacology: study the interaction between the organism and the drug and its variability
  - pharmacokinetics (PK) and pharmacodynamics (PD)

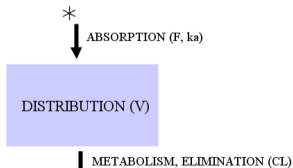
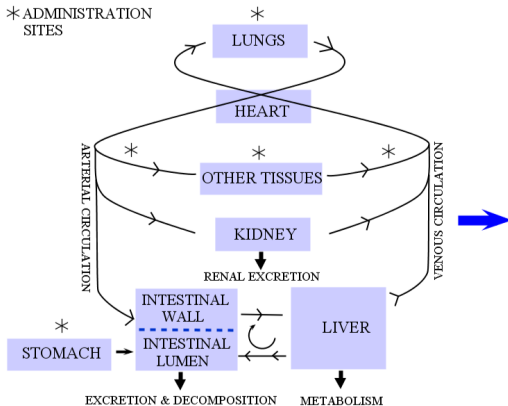
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- Pharmacogenetics (PG): genetic part of the variability
  - stratified medicine
- Genes coding for proteins involved in PK/PD processes
  - metabolism enzymes (CYP450, NAT)
  - single nucleotide polymorphism (SNP)

# Modelling in pharmacology

- Semi-physiological models integrating the *a priori* knowledge on the drug
  - parameters characterizing each physiological processes
  - model nonlinear in its parameters



$$\begin{aligned}
 Q_G(t=0) &= F \times \text{Dose} \\
 Q_P(t=0) &= 0 \\
 dQ_G/dt &= -k_a \times Q_G \\
 dQ_P/dt &= k_a \times Q_G - CL/V \times Q_P \\
 C_P(t) &= \frac{Q_P(t)}{V} \\
 &= \frac{F \times \text{Dose}}{V} \times \frac{k_a}{k_a - CL/V} (e^{-CL/V \times t} - e^{-k_a \times t})
 \end{aligned}$$

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  - parameter decomposed in fixed and random effects
  - ↪ covariates identification

Example :  $CL_i = CL + \beta \times SNP_i + \eta_i$  with  $SNP = \{0, 1, 2\}$

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- Estimation methods
  - Maximum likelihood: model linearization, Gaussian quadrature, SAEM
  - Bayesian inference (Lunn et al. 2002)



# Methodological challenges in PGx

- PK/PD phenotype → not observed
  - data : plasma or insulin concentrations, ...
  - ↪ dynamical models
- Variable informativeness of genetic markers
  - uneven distribution, small sample size of some genotypes
  - ↪ mixed effect models
- Increased size of the genetic data sets toward high throughput screening
  - dimensionality curse  $N \ll p$
  - structural correlation along the genome (linkage disequilibrium)
  - ↪ statistical genetics

# Genetic association analyses in PGx

- Stepwise procedure
  - commonly used for covariate model building
  - Lehr et al. (2010) adaptation for high throughput screening
- Integrated approach based on penalized regression
  - penalized regression established in animal and plant genetics
  - Lasso (Tibshirani. 1996)
  - HLasso (Hoggart et al. 2008)
    - developed for genome-wise association studies
    - higher effect size once included in the model
- Bayesian variable selection (O'Hara & Sillanpaa 2009)

# Stepwise procedure (Lehr et al. 2010)

$$y = \begin{bmatrix} y_{11} \\ y_{ij} \\ y_{Nn_N} \end{bmatrix}$$

$$SNP = \begin{bmatrix} SNP_{11} & SNP_{1s} & SNP_{1N_s} \\ SNP_{i1} & SNP_{is} & SNP_{iN_s} \\ SNP_{N1} & SNP_{Ns} & SNP_{NN_s} \end{bmatrix}$$

# Stepwise procedure (Lehr et al. 2010)

Basic dynamic

model:  $f(\phi_i, t_{ij})$

$\phi_i = [ka_i, Cl_i, \dots]$

$\phi_{pi} = \mu_p + \eta_{pi}$

$\eta_{pi} \sim N(0, \omega_p)$

Error model:

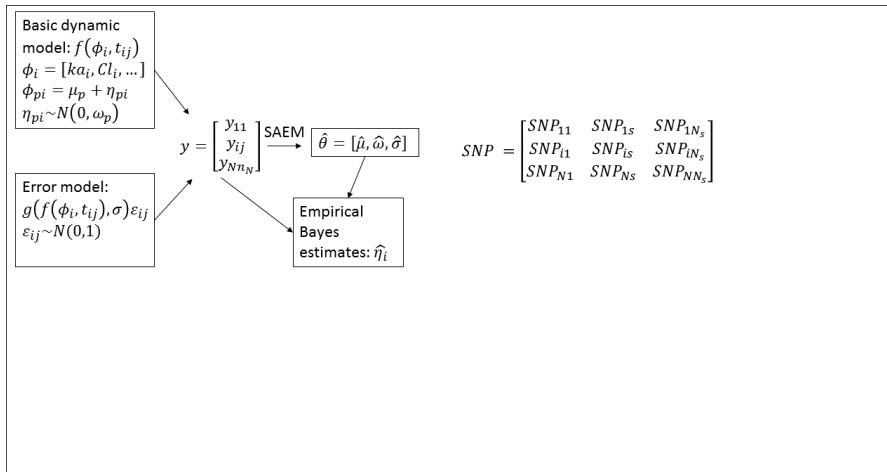
$g(f(\phi_i, t_{ij}), \sigma)\varepsilon_{ij}$

$\varepsilon_{ij} \sim N(0, 1)$

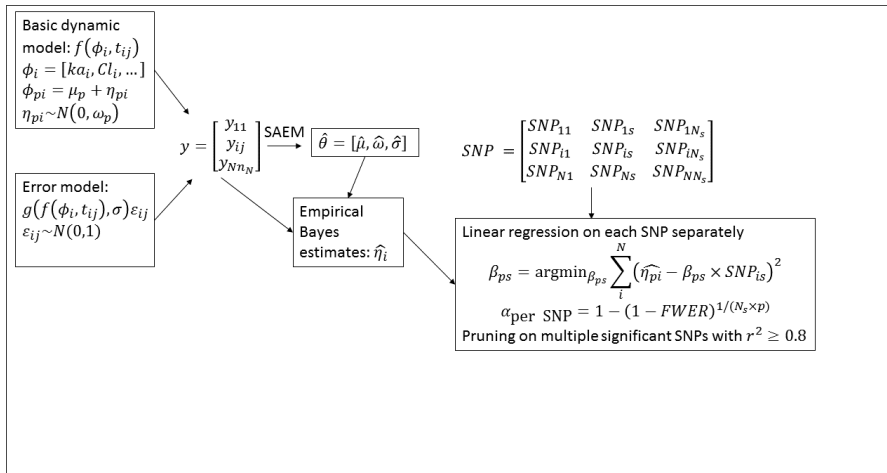
$$y = \begin{bmatrix} y_{11} \\ y_{ij} \\ y_{NnN} \end{bmatrix} \xrightarrow{\text{SAEM}} \hat{\theta} = [\hat{\mu}, \hat{\omega}, \hat{\sigma}]$$

$$SNP = \begin{bmatrix} SNP_{11} & SNP_{1s} & SNP_{1N_s} \\ SNP_{i1} & SNP_{is} & SNP_{iN_s} \\ SNP_{N1} & SNP_{Ns} & SNP_{NN_s} \end{bmatrix}$$

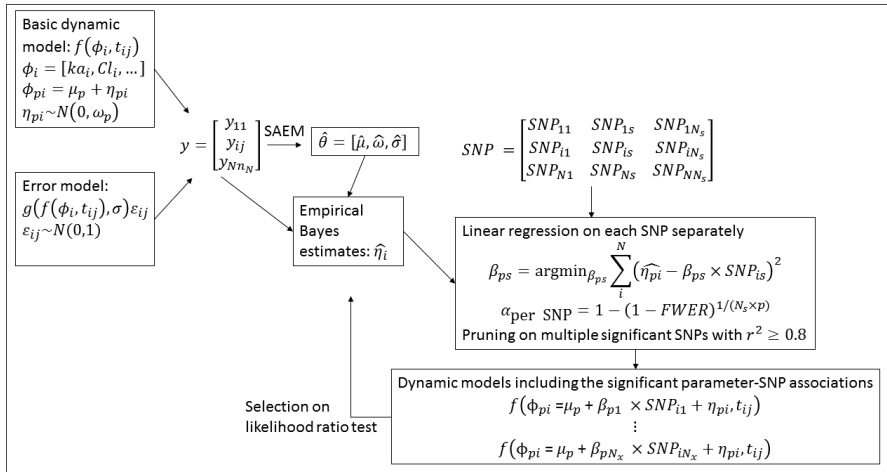
# Stepwise procedure (Lehr et al. 2010)



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↪ SNP selection after estimation of model parameters

↪ SNP considered independently

# Integrated appr. (Bertrand et al. 2015)

$$y = \begin{bmatrix} y_{11} \\ y_{ij} \\ y_{Nn_N} \end{bmatrix}$$

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# Integrated appr. (Bertrand et al. 2015)

Dynamic model including parameter-SNP associations:

$$f(\phi_i = \mu + \beta \times \text{SNP}_i + \eta_i, t_{ij})$$

$$\eta_i \sim N(0, \Omega)$$

Error model:

$$g(f(\phi_i, t_{ij}), \sigma) \varepsilon_{ij}$$

$$\varepsilon_{ij} \sim N(0, 1)$$

$$y = \begin{bmatrix} y_{11} \\ y_{ij} \\ y_{NnN} \end{bmatrix}$$

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SAEM

**Stochastic Approximation Expectation-step at iteration k**

$\phi'_{ik}$  drawn from  $p(\cdot | y; \theta_k)$ , Metropolis Hasting algorithm

$s_{ik} = s_{ik} + \tau_k (\phi'_{ik} - s_{ik-1})$ ,  $\tau_k$  sequence of decreasing positive numbers

**Maximization-step of  $\mu$  and  $\beta$  at iteration k**

$$(\widehat{\mu}, \widehat{\beta}) = \underset{\mu, \beta}{\text{argmin}} \sum_{i=1}^N (s_{ik} - \mu - \beta \times \text{SNP}_i) \Omega^{-1} (s_{ik} - \mu - \beta \times \text{SNP}_i)$$

$$\hat{\theta} = [\hat{\mu}, \hat{\beta}, \hat{\omega}, \hat{\sigma}]$$

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Error model:

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SAEM **modified**

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Lasso:  $P_{\xi}(\beta)$  approx. a double exponential prior on  $\beta$

Hlasso:  $P_{\gamma, \lambda}(\beta)$  approx. a normal exponential gamma prior on  $\beta$

$\xi$  and  $\gamma$  set using an asymptotic approximation

$$\hat{\theta} = [\hat{\mu}, \hat{\beta}, \hat{\omega}, \hat{\sigma}]$$

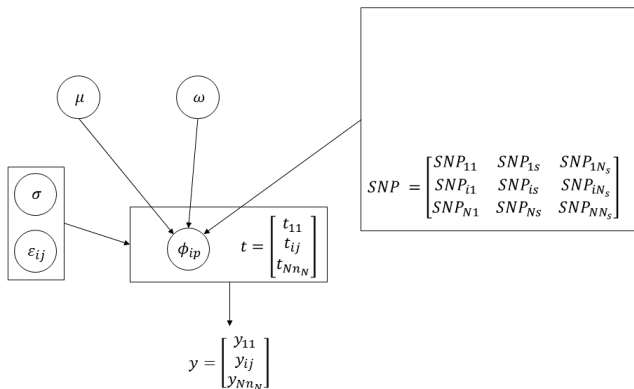
- ↪ Simultaneous SNP selection and estimation of model parameters
- ↪ All parameter-SNP associations considered simultaneously

# Bayesian approach (Kuo & Mallick 1998)

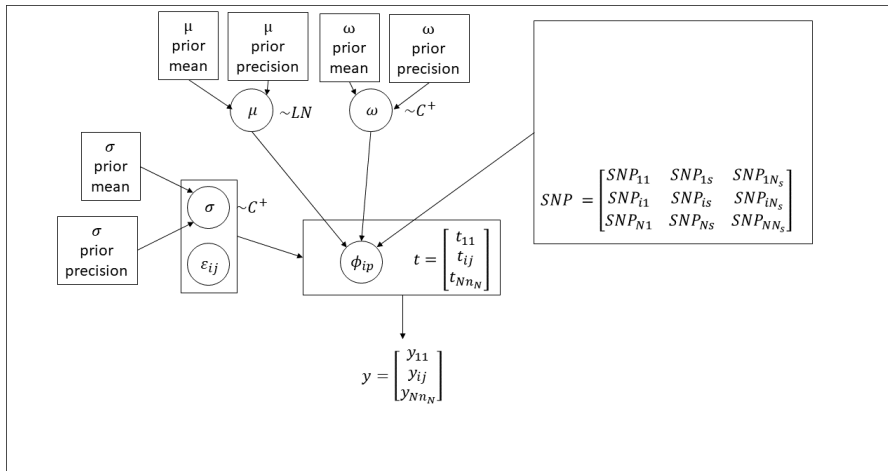
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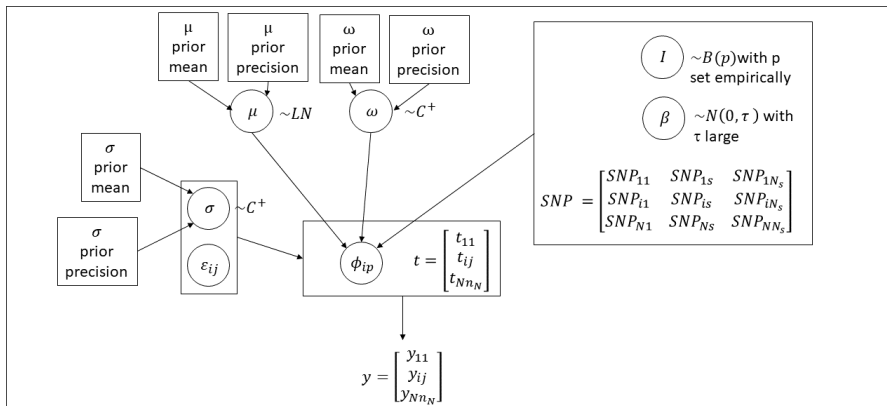
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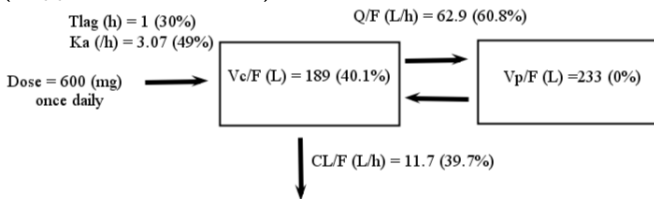
# Objectives

- To evaluate, through a realistic simulation study, the performance to detect a pharmacogenetic association
  - stepwise procedure
  - integrated approach with penalized regression
  - bayesian approach



## Genetic and PK data

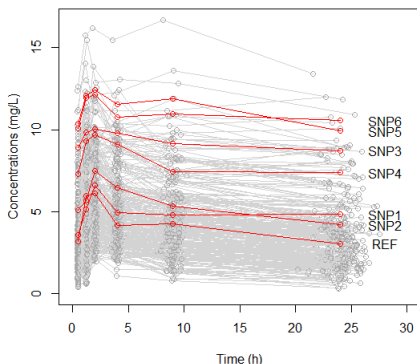
- Generation of genotypes using HAPGEN (Su et al. 2011)
  - $N_s=1227$  snps on 171 genes from the DMET Chip (Daly et al. 2007)
  - 6 [1-56] snps per gene
  - HAPMAP caucasian reference haplotypes
- Pharmacokinetic profiles inspired from real study (Kappelhoff et al. 2005)



- phase II study:  $N=300/t=0.5, 1.25, 2, 4, 9, 24h$
- diagonal variance matrix of random effects
- combined residual error model  $g = \sigma_{inter} + \sigma_{prop} f(\phi_i, t_{ij})$

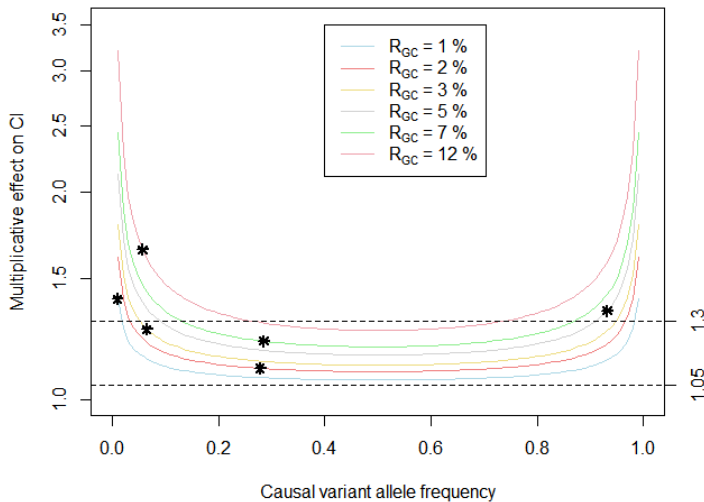
# Pharmacogenetic effect - 1/2

- 6 unobserved causal variants explaining 30% of the PK

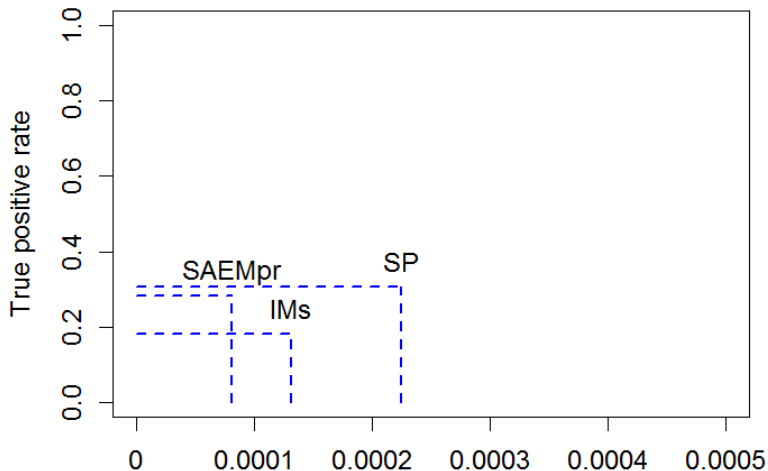


- $\log C_{I_i} = \log C_I + \sum_{s=1}^6 \beta_{C_{I_s}} SNP_{s_i} + \eta_{C_{I_i}}$  with  $SNP_{s_i} = 0, 1, 2$  i.e. allele dosage model

# Pharmacogenetic effect - 2/2



# True and false positive rates



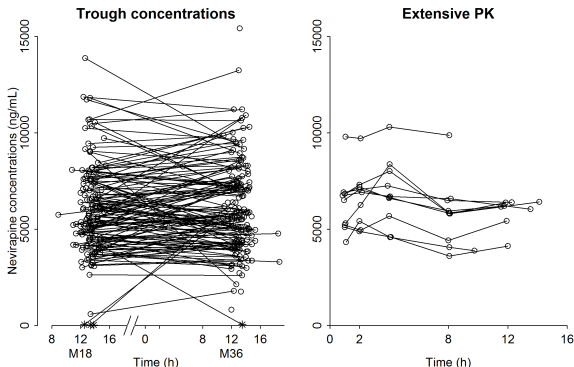
# Computing times in hours - mean [range]

<b>Stepwise procedure</b>	<b>0.24 [0.06 - 1.09]</b>
<b>Integrative appr.</b>	<b>1.14[0.83 - 1.61]</b>
<b>Bayesian appr.</b>	<b>19.58 [11.51 - 23.12]</b>

# Conclusions

- Feasibility of model-based PGx analyses on a real-case scenario
  - real need of increased sample size compared to classical drug development study designs
- Similar TPR between Integrative appr. and stepwise procedure
  - cost in computing time non-negligible
  - less so on larger data sets (not shown today)
  - better performance when multiple parameter-SNP associations (not shown today)
- Bayesian appr. performance not yet competitive
  - other indicator-based selection and/or shrinkage priors
  - use of Hamiltonian Monte Carlo (rstan package)

# PECAN ANRS 12154 (Dr AM Taburet and Prof D Haas)



- 129 patients on up to 3 occasions with 196 markers

Chromosome	3	7			19	
Gene	<i>NR1I2</i> (PXR)	<i>ABCB1</i> (P-gp)	<i>CYP3A5</i>	<i>CYP3A4</i>	<i>CYP2A6</i>	<i>CYP2B6</i>
Number of markers	49	63	1	36	1	47

- 218 missing polymorphisms with a maximum of 7 per subject

# Analyses

- One compartment model with 1st-order absorption and elimination
- Inter-individual and inter-occasion variability on CL
- 1 Stepwise procedure on empirical Bayes estimates with a priori adjustment for rs3745274
  - missing data removed
- 2 Indicator Model selection in JAGS
  - missing data imputed from Binomial with empirical allele frequency
- 2 Shrinkage prior in rstan
  - missing data imputed beforehand to most common genotype



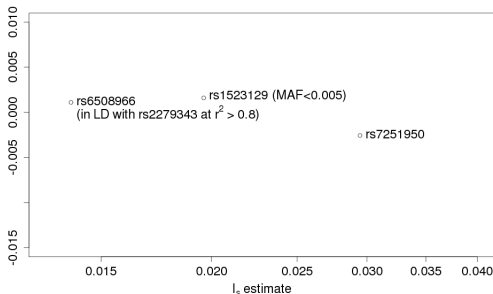
# Stepwise procedure (Bertrand et al. 2012)

**Table 3 Multivariate linear regression for association between genetic variants and nevirapine clearance estimate**

Variants	Gene	$\beta$	Statistic	<i>P</i> value
<i>n</i> = 129 (all participants)				
rs3745274	<i>CYP2B6</i>	-1.606	-6.66	$1.01 \times 10^{-9}$
rs2687116	<i>CYP3A4</i>	-1.913	-4.68	$7.95 \times 10^{-6}$
rs2279343	<i>CYP2B6</i>	0.904	4.13	$7.07 \times 10^{-5}$
rs7251950	<i>CYP2B6</i>	-0.499	-4.23	$4.77 \times 10^{-5}$
<i>n</i> = 128 (without outlier) <sup>a</sup>				
rs3745274	<i>CYP2B6</i>	-1.568	-7.66	$6.78 \times 10^{-12}$
rs2279343	<i>CYP2B6</i>	0.835	4.50	$1.66 \times 10^{-5}$
rs7251950	<i>CYP2B6</i>	-0.404	-4.01	$1.09 \times 10^{-4}$
rs2032582A	<i>ABCB1</i>	0.456	3.60	$4.73 \times 10^{-4}$

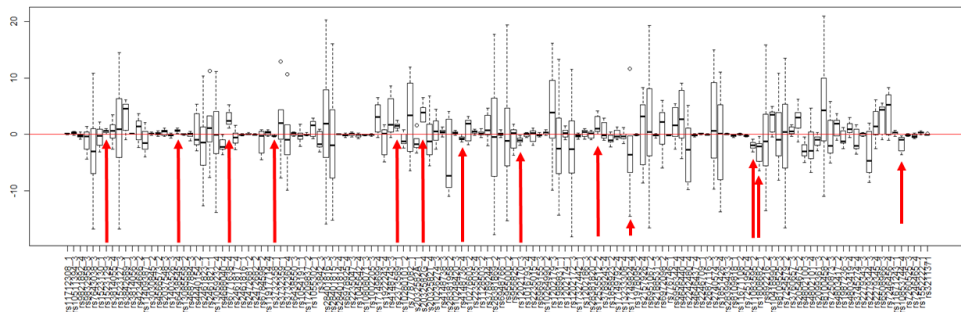
<sup>a</sup>Data from one individual outlier (extremely high) nevirapine clearance value were censored from this analysis.

# Indicator Model selection in JAGS (Kuo & Mallick 1998)



- $\beta = I \times \beta_s$   
with  $\beta_s \sim N(0, 1)$ ,  $I \sim B(p_{ind})$  and  $p_{ind} \sim Beta(.5, .5)$
- 3 chains / nburn=200 / niter=2000 / Neff = [15 -750] / approx. 5 min

# Shrinkage prior in rstan

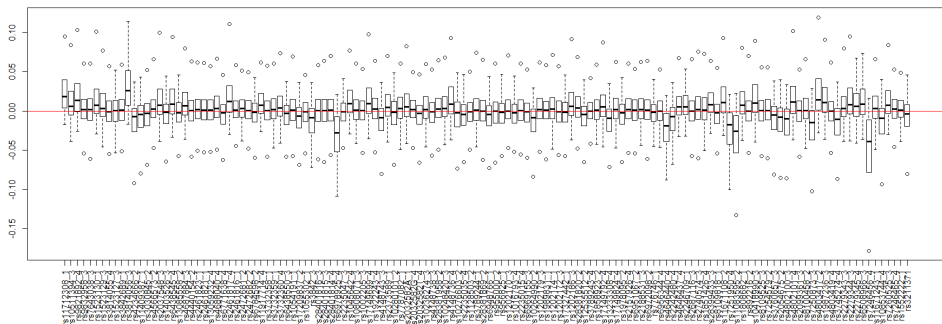


- normal prior (Hoerl & Kennard 1970)

$$\beta \sim N(0, 10)$$

- 3 chains / nburn=300 / niter=900 / Neff = [81 - 663] / approx. 20 min

# Shrinkage prior in rstan

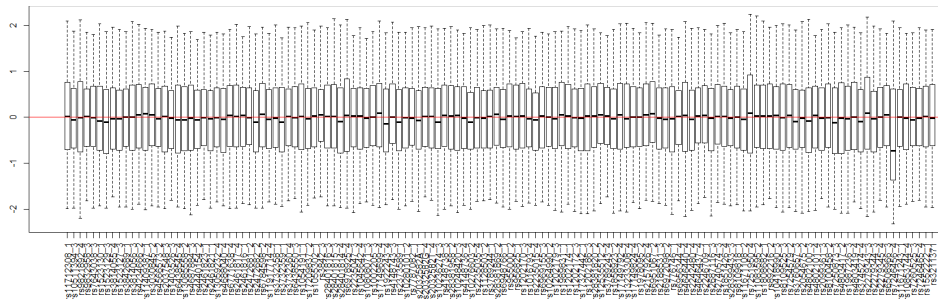


- Lasso prior (Tibshirani 1996)

$$\beta \sim N(0, \tau) \text{ with } \tau \sim C^+(0, .01)$$

- 3 chains / nburn=300 / niter=900 / Neff = [81 - 663] / approx. 20 min

# Shrinkage prior in rstan



- Horseshoe prior (Peltola et al. 2014)

$$\beta \sim N(0, \lambda_i \tau) \text{ with } \lambda_i \sim C^+(0, 1) \text{ and } \tau \sim U(0, 1)$$

- 3 chains / nburn=300 / niter=900 / Neff = [81 - 663] / approx. 20 min

# Conclusion

- Stepwise approach and Bayesian variable selection in JAGs detect similar signals
- Rstan sampling marginally more efficient than JAGs
  - too strong shrinkage of Lasso and Horseshoe ?
- Work in progress with yet a lot to do...
  - run rstan on simulation study
  - investigate how to set prior parameters
  - application to HIV Swiss cohort and EARSII trial on diabetes data

# Acknowledgements

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