Deriving informative prior distributions by the combination of the opinions of several experts in a Bayesian hierarchical approach. An application in radiation epidemiology.

Sabine Hoffmann[1], Sophie Ancelet[1],
Chantal Guihenneuc[2]

[1]IRSN PRP-HOM/SRBE/Laboratoire d'épidémiologie (LEPID)
[2] Université Paris Descartes

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Background

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⇒ Elicit expert opinion to derive prior distributions

⇒ Challenges:
  - Develop an elicitation procedure that allows to derive a prior distribution reflecting an expert's knowledge
  - Combine the information of several experts to derive a unique prior distribution

Strategies to avoid cognitive biases in the elicitation process

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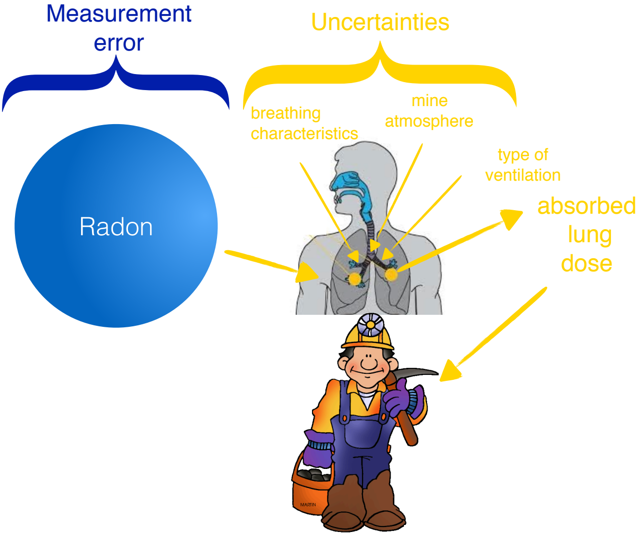
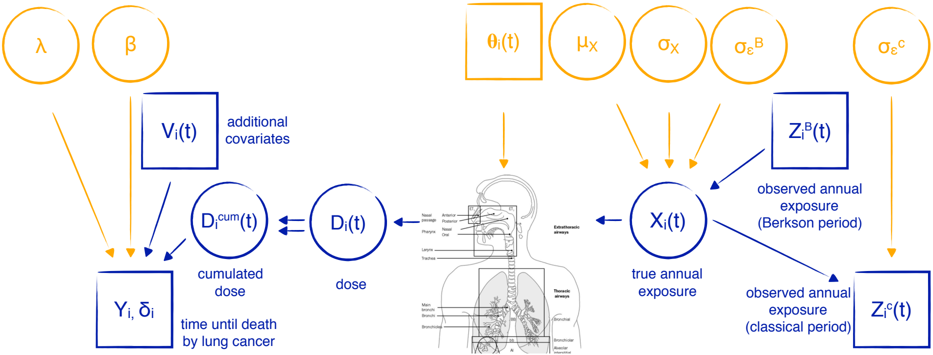
  - Quantiles are easier to assess than variances
- Difficult to give direct information concerning parameter uncertainty
  $\Rightarrow$ It may be easier to derive information based on the expert's choices
- Experts should be given a training in the elicitation procedure
- Give visual feedback
- Assess the confidence an expert has in his answers

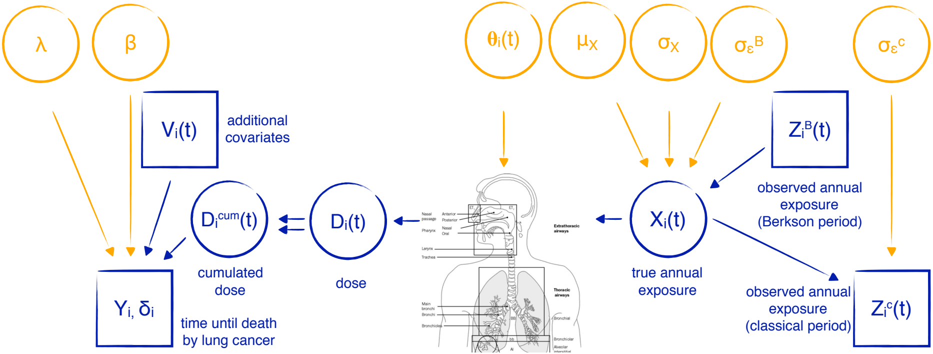
Motivating example: Dose estimation in uranium miners



Accounting for exposure uncertainty



Integrating dose uncertainties



Uncertain quantities intervening in dose calculation

Sensitivity analyses show that the most important parameters intervening in dose calculation are:

- Biological inter-subject variability:
  - Breathing rate
  - Fraction breathed through nose
  - Thickness of the bronchial epithelium
- Activity size distribution
- Unattached fraction

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Breathing rate

- Average breathing rate of miner i at time t can be determined by multiplying the proportion of time a miner spent in a certain activity by the corresponding value for breathing rate:
$\bar{br}_i(t) = p_1(i,t) \cdot br_1 + p_2(i,t) \cdot br_2 + p_3(i,t) \cdot br_3$

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- Reference values commonly used in radiation protection are based on data on six miners in a metal mine in Tadjikistan and on 620 miners in a gold mine in South Africa [Birchall and James, 1994, Ruzer et al., 1995]

  - ICRP publication 66:
    1.0 - 1.7 m^3h^{-1}
  - Alpha risk project:
    0.9 - 1.7 m^3h^{-1}
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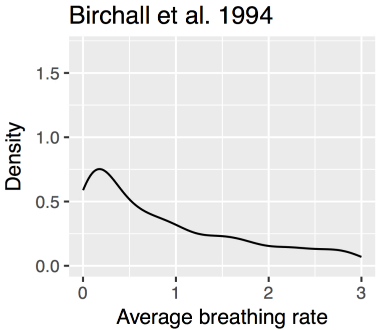

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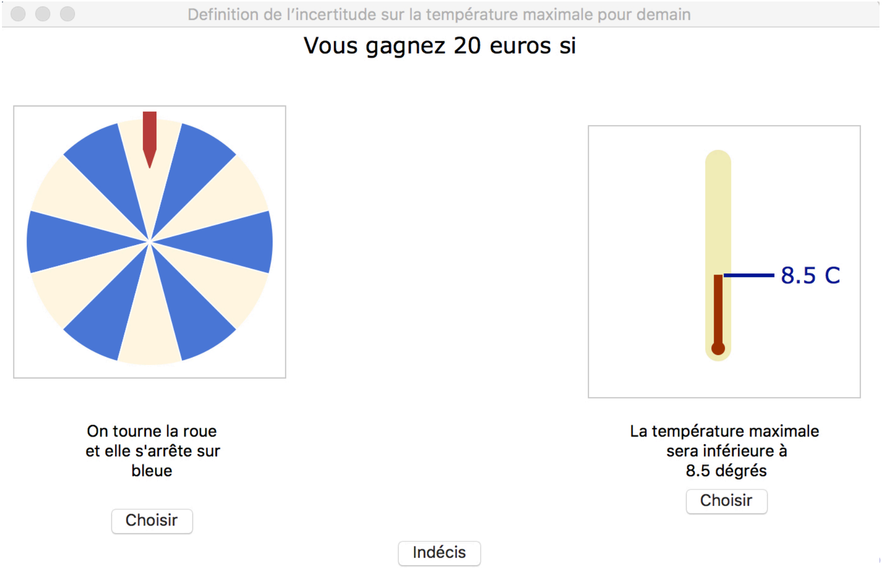
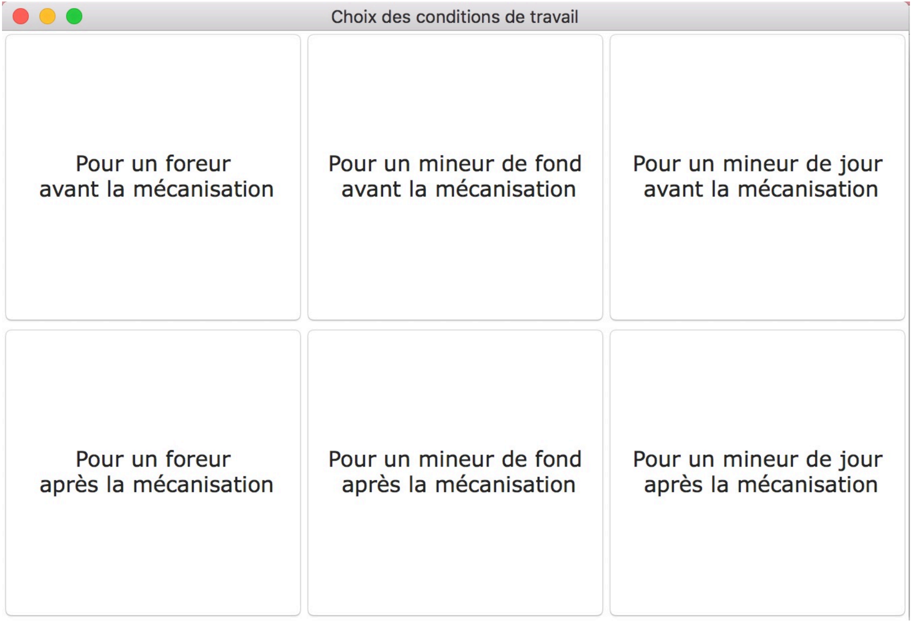
$\Rightarrow$ Ask experts about the time a miner spent sitting, in light activity and in heavy activity given a workday of eight hours

Six conditions: Hewers, underground and open pit miners and before and after mechanisation of the mines.

The Elicitation procedure

The training phase



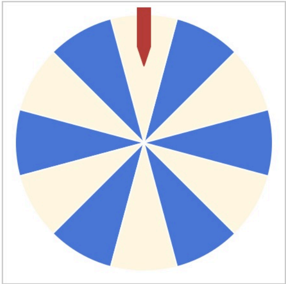

Choix des conditions de travail
Pour un foreur avant la mécanisation
Pour un mineur de fond avant la mécanisation
Pour un mineur de jour avant la mécanisation
Pour un foreur après la mécanisation
Pour un mineur de fond après la mécanisation
Pour un mineur de jour après la mécanisation

The elicitation procedure

- For each condition, the expert e ($e \in \{1, 2, 3\}$) is presented with a series of choices concerning the time a worker in this condition spent sitting, in light exercise and in heavy exercise

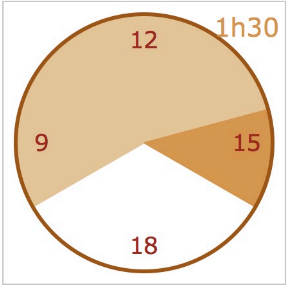
Definition de l'incertitude sur le temps assis pour un mineur de fond

Vous gagnez 20 euros si



On tourne la roue
et elle s'arrête sur
bleue

Choisir



Le temps assis pour un
mineur de fond
après la mécanisation
est inférieur à
1 heures et 30 minutes

Choisir

Indécis

Definition de l'incertitude sur le temps assis pour un mineur de fond

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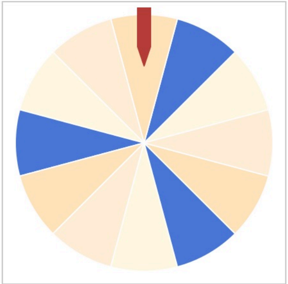
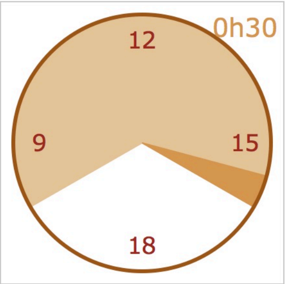


On tourne la roue et elle s'arrête sur bleue

Choisir



Le temps assis pour un mineur de fond après la mécanisation est inférieur à 0 heures et 30 minutes

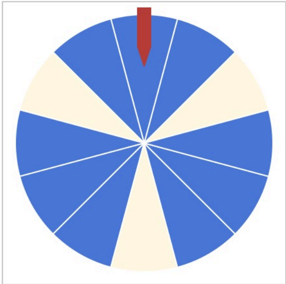
Choisir

Indécis

Revenir en arrière

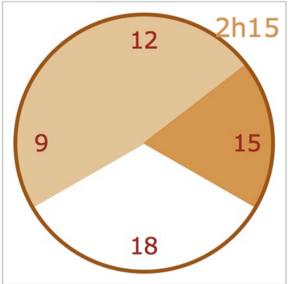
Definition de l'incertitude sur le temps assis pour un mineur de fond

Vous gagnez 20 euros si



On tourne la roue
et elle s'arrête sur
bleue

Choisir



Le temps assis pour un
mineur de fond
après la mécanisation
est inférieur à
2 heures et 15 minutes

Choisir

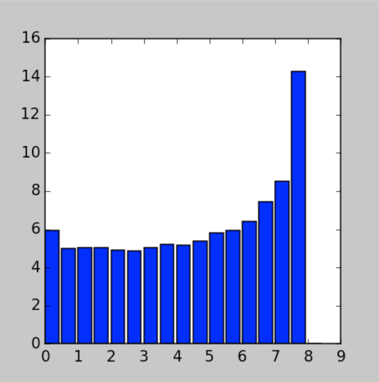
Indécis

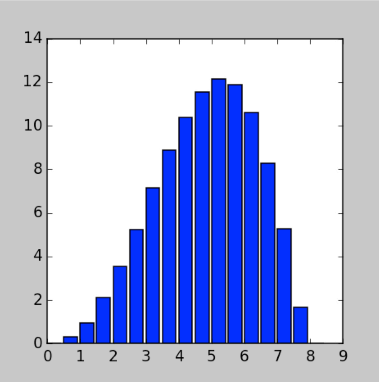
The elicitation procedure

- For each condition, the expert e ($e \in \{1, 2, 3\}$) is presented with a series of choices concerning the time a worker in this condition spent sitting, in light exercise and in heavy exercise
- Based on these binary choices, we derive the first quartile $q_{ej}^{0.25}$, the median $q_{ej}^{0.5}$ and the third quartile $q_{ej}^{0.75}$ of expert e and variable j, $j \in \{1, 2, 3\}$
- Using an interval-halving algorithm and a least squares method, we fit two alternative beta distributions based on $q_{ej}^{0.25}$ and $q_{ej}^{0.5}$, and $q_{ej}^{0.75}$ and $q_{ej}^{0.5}$, respectively.

Evaluation de la confiance en vos informations concernant foreur après la mécanisation

Quelle confiance avez vous dans votre choix pour le temps avec une forte activité physique ?





Evaluation

0 1 2 3 4 5 6 7 8 9

Recommencer

Choisir une autre variable

Evaluation

0 1 2 3 4 5 6 7 8 9

The elicitation procedure

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- The expert is presented with two alternative histograms representing the two fitted Beta distributions
- He expresses his confidence in these distributions via $c_{ej}^{0.25}$ and $c_{ej}^{0.75}$ taking values between 1 and 9.

Fitting a distribution that reflects an expert's knowledge

- For each condition, we dispose of 9 quantiles and 6 evaluations for an expert e at the end of the elicitation process:
  - Sitting: $q_{e1}^{0.25}, q_{e1}^{0.5}, q_{e1}^{0.75}$ with evaluations c_{e1}^{1}, c_{e1}^{2}
  - Light exercise: $q_{e2}^{0.25}, q_{e2}^{0.5}, q_{e2}^{0.75}$ with evaluations c_{e2}^{1}, c_{e2}^{2}
  - Heavy exercise: $q_{e3}^{0.25}, q_{e3}^{0.5}, q_{e3}^{0.75}$ with evaluations c_{e2}^{1}, c_{e3}^{2}

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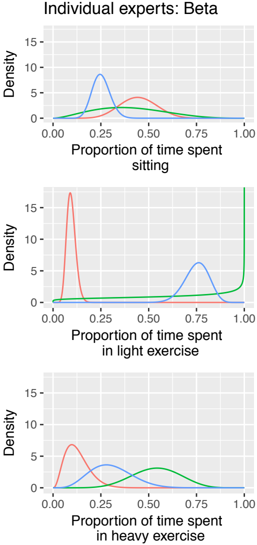
$\Rightarrow$ Idea 1:

Fit independent beta distributions with parameters α_{ej} and β_{ej} via least squared for each variable j, denoted P_{ej} hereafter

Results for a hewer after the mechanisation



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$\Rightarrow$ Idea 2:

Fit marginal beta distributions of a Dirichlet distribution with parameters $\hat{a}_{e1}, \hat{a}_{e2}$ and $\hat{a}_{e3}$ so that $P_{e1} + P_{e2} + P_{e3} = 1$

- $P_{e1} \sim Beta(\hat{a}_{e1}, \hat{a}_{e2} + \hat{a}_{e3})$
- $P_{e2} \sim Beta(\hat{a}_{e2}, \hat{a}_{e1} + \hat{a}_{e3})$
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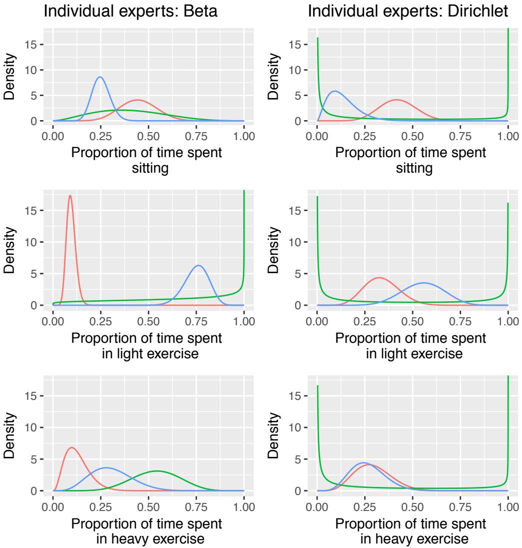

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- Denote f_e the Dirichlet distribution with parameters $\hat{a}_{e1}, \hat{a}_{e2}$ and $\hat{a}_{e3}$ derived for expert e with relative weight $\pi_e = \frac{\sum_{k,l} c_{ek}^l}{\sum_{i,k,l} c_{ik}^l}$

Combining the information of several experts

Approaches for the aggregation of expert knowledge

- Averaging
- Mixture modeling
- Hierarchical approach

Averaging

Given

- f_1 = Dirichlet($\hat{a}_{11}, \hat{a}_{12}, \hat{a}_{13}$), π_1
- f_2 = Dirichlet($\hat{a}_{21}, \hat{a}_{22}, \hat{a}_{23}$), π_2
- f_3 = Dirichlet($\hat{a}_{31}, \hat{a}_{32}, \hat{a}_{33}$), π_3

derive an averaged distribution f = Dirichlet(a_1, a_2, a_3) with

- $a_1 = \pi_1 \hat{a}_{11} + \pi_2 \hat{a}_{21} + \pi_3 \hat{a}_{31}$
- $a_2 = \pi_1 \hat{a}_{12} + \pi_2 \hat{a}_{22} + \pi_3 \hat{a}_{32}$
- $a_3 = \pi_1 \hat{a}_{13} + \pi_2 \hat{a}_{23} + \pi_3 \hat{a}_{33}$

Mixture modeling

Given

- $f_1 = \text{Dirichlet}(\hat{a}_{11}, \hat{a}_{12},\ \hat{a}_{13}),\ \pi_1$
- $f_2 = \text{Dirichlet}(\hat{a}_{21}, \hat{a}_{22}, \hat{a}_{23}),\ \pi_2$
- $f_3 = \text{Dirichlet}(\hat{a}_{31}, \hat{a}_{32}, \hat{a}_{33}),\ \pi_3$

derive the mixture model $f = \pi_1 f_1 + \pi_2 f_2 + \pi_3 f_3$

Hierarchical approach

- Derive $f = Dirichlet(a_1, a_2, a_3)$ with $a_1 = s\theta_1, a_2 = s\theta_2$ and $a_3 = s - a_1 - a_2$
- Treat the parameters of the fitted Dirichlet distributions as data
- Reparameterize: $\hat{s}_e = \hat{a}_{e1} + \hat{a}_{e2} + \hat{a}_{e3}$, $\hat{\theta}_{e1} = \frac{\hat{a}_{e1}}{\hat{s}_e}$ and $\hat{\theta}_{e2} = \frac{\hat{a}_{e2}}{\hat{s}_e}$
- Model $\hat{\theta}_{e1}, \hat{\theta}_{e2}$ and $\hat{s}_e$ as
  - $\hat{\theta}_{e1}|\theta_1, s, \pi_e \sim Beta(\theta_1 s\pi_e, (1-\theta_1)s\pi_e)$
  - $\hat{\theta}_{e2}|\hat{\theta}_{e1}, \theta_2, s, \pi_e \sim Beta(\theta_2 s\pi_e, (1-\theta_2)s\pi_e)\mathbb{1}_{\hat{\theta}_{e2} \leq 1-\hat{\theta}_{e1}}$
  - $\hat{s}_e|s, \pi_e \sim Gamma(\pi_e, \pi_e/s)$
- Assume the following prior distributions:
  - $\theta_1 \sim Unif(0,1)$
  - $\theta_2|\theta_1 \sim Unif(0,1)\mathbb{1}_{\theta_2 \leq 1-\theta_1}$
  - $s \sim Exp(0.01)$

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  - $s \sim Exp(0.01)$

$\Rightarrow E(\hat{\theta}_{e1}) = \theta_1, E(\hat{\theta}_{e2}) = \theta_2, E(\hat{s}_e) = s$

$\Rightarrow Var(\hat{\theta}_{e1}) = \frac{\theta_1(1-\theta_1)}{s\pi_e+1}, Var(\hat{\theta}_{e2}) = \frac{\theta_2(1-\theta_2)}{s\pi_e+1}, Var(\hat{s}_e) = \frac{s^2}{\pi_e}$

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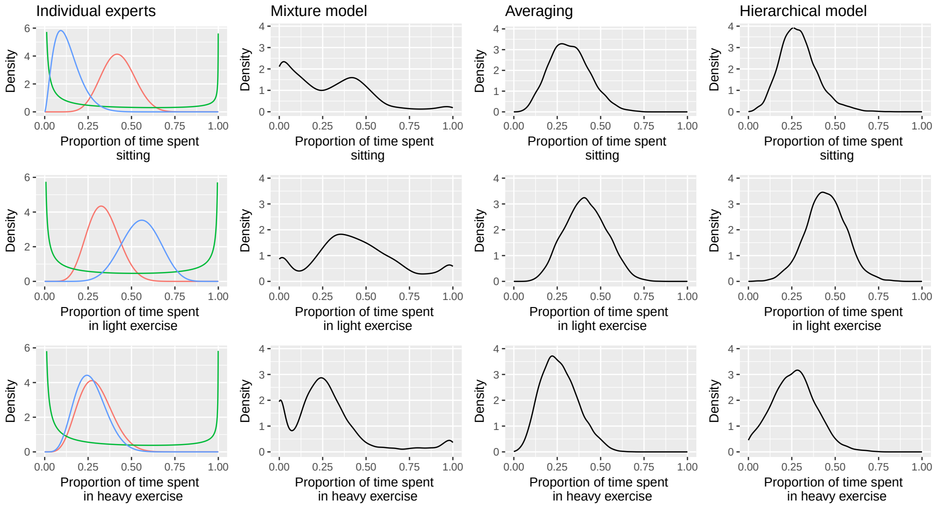
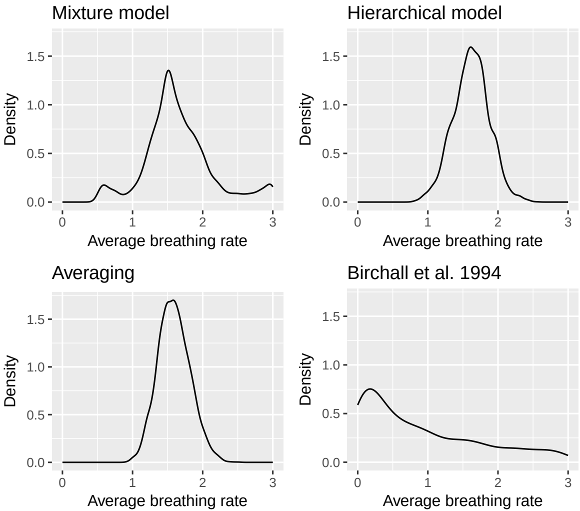


Derive a prior distribution on breathing rate

$$\bar{br}_i(t) = P_{1i}(t) \cdot br_1 + P_{2i}(t) \cdot br_2 + P_{3i}(t) \cdot br_3$$



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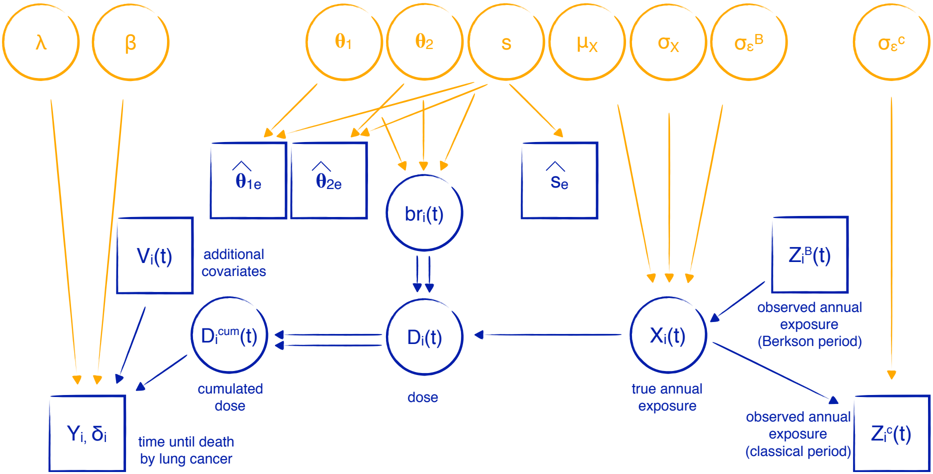
Perspectives

- The hierarchical approach for the aggregation of the opinion of several experts allows to combine the information derived by expert opinion with the information available in the literature
- Integrate the prior elicitation sub-model in the hierarchical model to account for exposure and dose uncertainty

Perspectives



Thank you for your attention

Birchall, A. and James, A. C. (1994).
Uncertainty analysis of the effective dose per unit exposure from radon progeny and implications for icrp risk-weighting factors.
Radiation Protection Dosimetry, 53(1-4):133–140.

Ruzer, L. S., V., N. A., and Harley, N. H. (1995).
Assessment of lung deposition and breathing rate of underground miners in tadjikistan.
Radiation Protection Dosimetry, 58:261–268.