Uncertainty quantification and calibration of a photovoltaic plant model: warranty of performance and robust estimation of the long-term production

Mathieu Carmassi
Thesis director : Eric Parent
Advisors : Pierre Barbillon, Matthieu Chiodetti, Merlin Keller

EDF - UMR MIA Paris AgroParisTech/INRA

13^{th} December, 2018

Introduction

Context

Industrial context

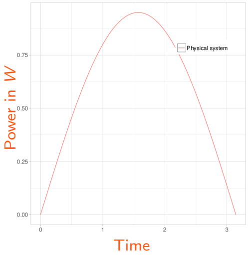
A certain amount of experimental data is gathered on the field.

Introduction

Context

Industrial context

A certain amount of experimental data is gathered on the field.

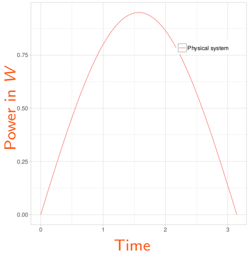


Introduction

Context

Industrial context

A certain amount of experimental data is gathered on the field.

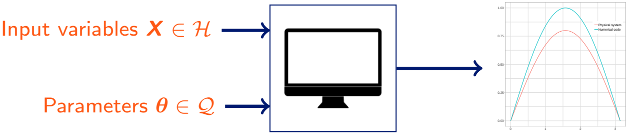


Industrial issues

How to trust the numerical code which, supposedly reproduces the experimental data?

Introduction

Context

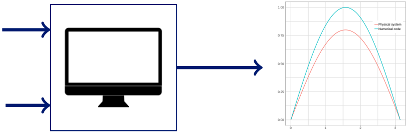


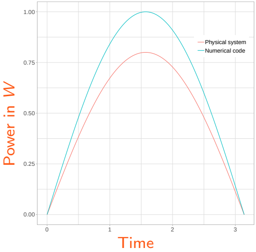
Introduction

Context

Input variables $\boldsymbol{X} \in \mathcal{H}$

Parameters $\boldsymbol{\theta} \in \mathcal{Q}$



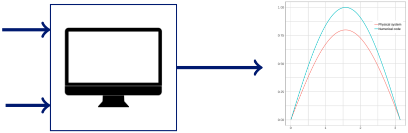


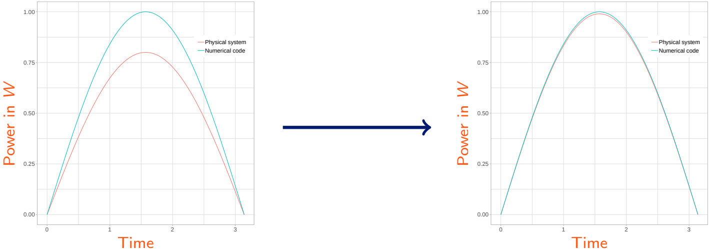
Introduction

Context

Input variables $\boldsymbol{X} \in \mathcal{H}$

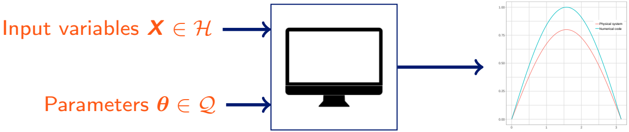
Parameters $\boldsymbol{\theta} \in \mathcal{Q}$





Introduction

Context

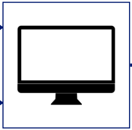


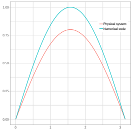
Introduction

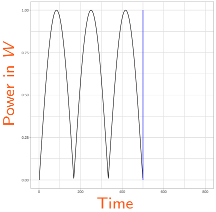
Context

Input variables $\boldsymbol{X} \in \mathcal{H}$

Parameters $\boldsymbol{\theta} \in \mathcal{Q}$





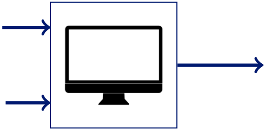


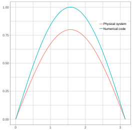
Introduction

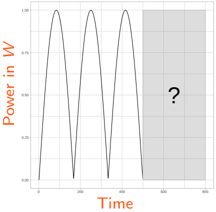
Context

Input variables $\boldsymbol{X} \in \mathcal{H}$

Parameters $\boldsymbol{\theta} \in \mathcal{Q}$





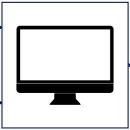


Introduction

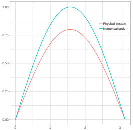
Context

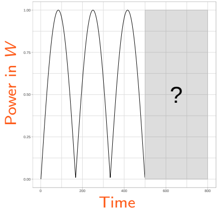
Input variables $\boldsymbol{X} \in \mathcal{H}$ →

Parameters $\boldsymbol{\theta} \in \mathcal{Q}$ →



→

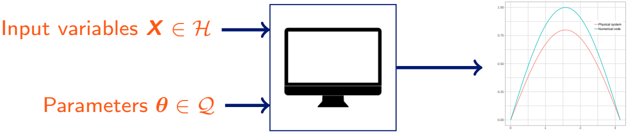


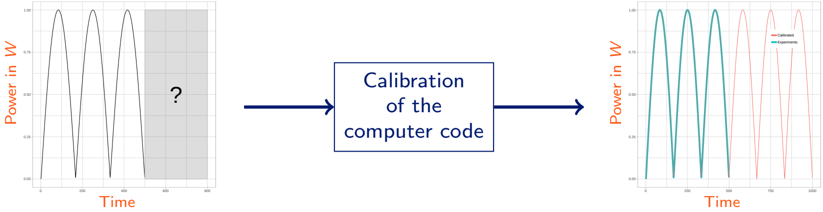


→ Calibration of the computer code

Introduction

Context





Contents

I

Calibration in theory

- Mathematical definition of the numerical code
- Different statistical models
- Estimation methods

II

Application Case:
Code for the prediction of power from a PV plant (Python code)

Contents

I

Calibration in theory

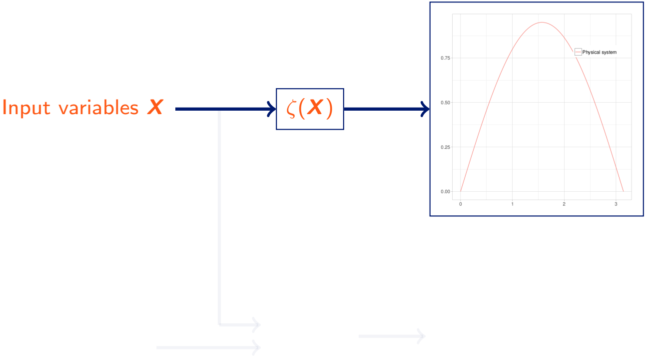
- **Mathematical definition of the numerical code**
- Different statistical models
- Estimation methods

II

Application Case:
Code for the prediction of power from a PV plant (Python code)

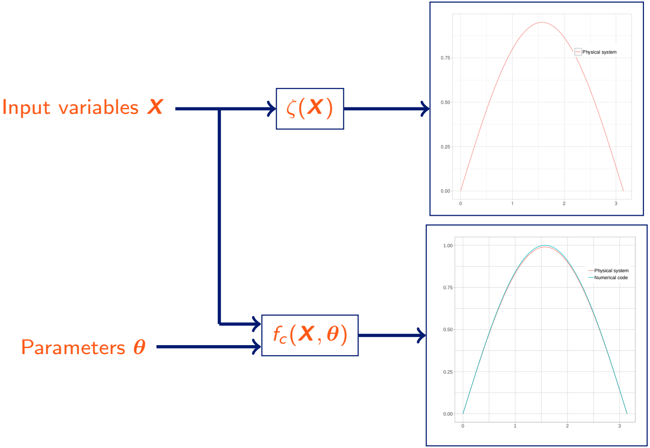
Calibration

Physical system and the code



Calibration

Physical system and the code



Calibration

First statistical model

The measurement error

$$\forall t \in \{1, \dots, T\} \quad Y_{exp_t} = \zeta(\boldsymbol{X}_t) + \epsilon_t, \tag{1}$$

where $\epsilon_t \overset{iid}{\sim} \mathcal{N}(0, \sigma^2_{err})$.

Calibration

First statistical model

The measurement error

$$\forall t \in \{1, \ldots, T\} \quad Y_{exp_t} = \zeta(\boldsymbol{X}_t) + \epsilon_t, \tag{1}$$

where $\epsilon_t \overset{iid}{\sim} \mathcal{N}(0, \sigma_{err}^2)$.

The first model

$$\mathcal{M}_1 \ : \ \forall t \in \{1, \ldots, T\} \quad Y_{exp_t} = f_c(\boldsymbol{x}_t, \boldsymbol{\theta}) + \epsilon_t. \tag{2}$$

Context

I

Calibration in theory

- Mathematical definition of the numerical code
- **Different statistical models**
- Estimation methods

II

Application Case:
Code for the prediction of power from a PV plant (Python code)

Calibration

Second statistical model

In the case of a time consuming code

In **Maximum Likelihood Estimation** (MLE) or in a **Bayesian estimation** (which recourse to many MCMC iteration)

$\Downarrow$

intractable to work with a time consuming f_c (Sacks et al., 1989)

Calibration

Second statistical model

In the case of a time consuming code

In **Maximum Likelihood Estimation** (MLE) or in a **Bayesian estimation** (which recourse to many MCMC iteration)

$\Downarrow$

intractable to work with a time consuming f_c (Sacks et al., 1989)

The second model (Cox et al., 2001)

$$\mathcal{M}_2 \ : \ \forall t \in \{1, \dots, T\} \quad Y_{exp_t} = \boldsymbol{F}(\boldsymbol{x}_t, \boldsymbol{\theta}) + \epsilon_t, \tag{3}$$

where $\boldsymbol{F}(\bullet, \bullet) \sim \mathcal{GP}\Big(m_c(\bullet, \bullet), c_c(\{\bullet, \bullet\}, \{\bullet, \bullet\})\Big)$.

Calibration

The Gaussian process

Mean

$$m_c(\bullet,\bullet) = \beta_{c_0} + \sum_{j=1}^{M} \beta_{c_j} h_{c_j}(\bullet,\bullet),$$

where $\boldsymbol{\beta}_c = (\beta_{c_1}, \ldots, \beta_{c_M})$ is the coefficient vector to be estimated, and $\boldsymbol{h}_c(\bullet,\bullet) = (h_{c_0}(\bullet,\bullet), \ldots, h_{c_M}(\bullet,\bullet))$ is the design matrix of regression.

Covariance

$$c_c(\{\bullet,\bullet\},\{\bullet,\bullet\}) = \sigma_c^2 r_{c_{\psi_c}}(\{\bullet,\bullet\},\{\bullet,\bullet\}),$$

where σ_c^2 stands for the variance, and
$r_{c_{\psi_c}}$ for the correlation function with a vector parameter ψ_c which is the scale and the regularity of the kernel.

Calibration

The third model

The discrepancy

Some papers as Bayarri et al. (2007),Kennedy and O'Hagan (2001),Higdon et al. (2004) advocate adding another error term:

$$\delta(\boldsymbol{x}_t) = \zeta(\boldsymbol{x}_t) - f_c(\boldsymbol{x}_t, \boldsymbol{\theta}^*).$$

Calibration

The third model

The discrepancy

Some papers as Bayarri et al. (2007),Kennedy and O'Hagan (2001),Higdon et al. (2004) advocate adding another error term:

$$\boldsymbol{\delta}(\boldsymbol{x}_t) = \zeta(\boldsymbol{x}_t) - f_c(\boldsymbol{x}_t, \boldsymbol{\theta}^*).$$

The third model

$$\mathcal{M}_3 \; : \; \forall t \in \{1, \dots, T\} \quad Y_{exp_t} = f_c(\boldsymbol{x}_t, \boldsymbol{\theta}) + \boldsymbol{\delta}(\boldsymbol{x}_t) + \epsilon_t, \tag{4}$$

where $\boldsymbol{\delta}(\boldsymbol{x}_t) \sim \mathcal{GP}(m_\delta(\bullet), c_\delta(\bullet, \bullet))$.

Calibration

The third model

The discrepancy

Some papers as Bayarri et al. (2007),Kennedy and O'Hagan (2001),Higdon et al. (2004) advocate adding another error term:

$$\boldsymbol{\delta}(\boldsymbol{x}_t) = \zeta(\boldsymbol{x}_t) - f_c(\boldsymbol{x}_t, \boldsymbol{\theta}^*).$$

The third model

$$\mathcal{M}_3 \ : \ \forall t \in \{1, \dots, T\} \quad Y_{exp_t} = f_c(\boldsymbol{x}_t, \boldsymbol{\theta}) + \boldsymbol{\delta}(\boldsymbol{x}_t) + \epsilon_t, \tag{4}$$

where $\boldsymbol{\delta}(\boldsymbol{x}_t) \sim \mathcal{GP}(m_\delta(\bullet), c_\delta(\bullet, \bullet))$.

Identifiability problem

An infinity of $(\boldsymbol{\theta}, \boldsymbol{\delta})$ is possible.

Calibration

The fourth model

The fourth model (Kennedy and O'Hagan, 2001)

$$\mathcal{M}_4 \ : \ \forall t \in \{1, \ldots, T\} \quad Y_{exp_t} = F(\boldsymbol{x}_t, \boldsymbol{\theta}) + \boldsymbol{\delta}(\boldsymbol{x}_t) + \epsilon_t. \qquad (5)$$

Calibration

The fourth model

The fourth model (Kennedy and O'Hagan, 2001)

$$\mathcal{M}_4 \ : \ \forall t \in \{1, \ldots, T\} \quad Y_{exp_t} = F(\boldsymbol{x}_t, \boldsymbol{\theta}) + \boldsymbol{\delta}(\boldsymbol{x}_t) + \epsilon_t. \tag{5}$$

Summary

the model	$\zeta(\boldsymbol{X})$
$\mathcal{M}_1$	$f_c(\boldsymbol{X}, \boldsymbol{\theta})$
$\mathcal{M}_2$	$\boldsymbol{F}(\boldsymbol{X}, \boldsymbol{\theta})$
$\mathcal{M}_3$	$f_c(\boldsymbol{X}, \boldsymbol{\theta}) + \delta(\boldsymbol{X})$
$\mathcal{M}_4$	$\boldsymbol{F}(\boldsymbol{X}, \boldsymbol{\theta}) + \delta(\boldsymbol{X})$

Context

I

Calibration in theory

- Mathematical definition of the numerical code
- Different statistical models
- **Estimation methods**

II

Application Case:
Code for the prediction of power from a PV plant (Python code)

Calibration

Likelihoods

Notations

- let us consider D a N sized DOE chosen in the space $\mathcal{H} \times \mathcal{Q}$ such that $\boldsymbol{y_c} = f_c(D)$,
- notations for the **surrogate**:
  - $\boldsymbol{H}_c$ the design matrix,
  - $\Phi_c = \{\beta_c, \psi_c, \sigma_c^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_c the correlation function,
- notations for the **discrepancy**:
  - $\boldsymbol{H}_\delta$ the design matrix,
  - $\Phi_\delta = \{\beta_\delta, \psi_\delta, \sigma_\delta^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_δ the correlation function,
- $\Phi = \{\Phi_c, \Phi_\delta\}$,
- $\boldsymbol{y} = (\boldsymbol{y}_{exp}, \boldsymbol{y}_c)^T$ of size $T + N$.

Calibration

Likelihoods

Notations

- let us consider D a N sized DOE chosen in the space $\mathcal{H} \times \mathcal{Q}$ such that $\boldsymbol{y_c} = f_c(D)$,
- notations for the **surrogate**:
  - $\boldsymbol{H}_c$ the design matrix,
  - $\Phi_c = \{\beta_c, \psi_c, \sigma_c^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_c the correlation function,
- notations for the **discrepancy**:
  - $\boldsymbol{H}_\delta$ the design matrix,
  - $\Phi_\delta = \{\beta_\delta, \psi_\delta, \sigma_\delta^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_δ the correlation function,
- $\Phi = \{\Phi_c, \Phi_\delta\}$,
- $\boldsymbol{y} = (\boldsymbol{y}_{exp}, \boldsymbol{y}_c)^T$ of size $T + N$.

Calibration

Likelihoods

Notations

- let us consider D a N sized DOE chosen in the space $\mathcal{H} \times \mathcal{Q}$ such that $\boldsymbol{y_c} = f_c(D)$,
- notations for the **surrogate**:
  - $\boldsymbol{H}_c$ the design matrix,
  - $\Phi_c = \{\beta_c, \psi_c, \sigma_c^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_c the correlation function,
- notations for the **discrepancy**:
  - $\boldsymbol{H}_\delta$ the design matrix,
  - $\Phi_\delta = \{\beta_\delta, \psi_\delta, \sigma_\delta^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_δ the correlation function,
- $\Phi = \{\Phi_c, \Phi_\delta\}$,
- $\boldsymbol{y} = (\boldsymbol{y}_{exp}, \boldsymbol{y}_c)^T$ of size $T + N$.

Calibration

Likelihoods

Notations

- let us consider D a N sized DOE chosen in the space $\mathcal{H} \times \mathcal{Q}$ such that $\boldsymbol{y_c} = f_c(D)$,
- notations for the **surrogate**:
  - $\boldsymbol{H}_c$ the design matrix,
  - $\Phi_c = \{\boldsymbol{\beta}_c, \boldsymbol{\psi}_c, \sigma_c^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_c the correlation function,
- notations for the **discrepancy**:
  - $\boldsymbol{H}_\delta$ the design matrix,
  - $\Phi_\delta = \{\boldsymbol{\beta}_\delta, \boldsymbol{\psi}_\delta, \sigma_\delta^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_δ the correlation function,
- $\Phi = \{\Phi_c, \Phi_\delta\}$,
- $\boldsymbol{y} = (\boldsymbol{y}_{exp}, \boldsymbol{y}_c)^T$ of size $T + N$.

Calibration

Likelihoods

Notations

- let us consider D a N sized DOE chosen in the space $\mathcal{H} \times \mathcal{Q}$ such that $\boldsymbol{y_c} = f_c(D)$,
- notations for the **surrogate**:
  - $\boldsymbol{H}_c$ the design matrix,
  - $\Phi_c = \{\boldsymbol{\beta}_c, \boldsymbol{\psi}_c, \sigma_c^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_c the correlation function,
- notations for the **discrepancy**:
  - $\boldsymbol{H}_\delta$ the design matrix,
  - $\Phi_\delta = \{\boldsymbol{\beta}_\delta, \boldsymbol{\psi}_\delta, \sigma_\delta^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_δ the correlation function,
- $\Phi = \{\Phi_c, \Phi_\delta\}$,
- $\boldsymbol{y} = (\boldsymbol{y}_{exp}, \boldsymbol{y}_c)^T$ of size $T + N$.

Calibration

Likelihoods

Notations

- let us consider D a N sized DOE chosen in the space $\mathcal{H} \times \mathcal{Q}$ such that $\boldsymbol{y_c} = f_c(D)$,
- notations for the **surrogate**:
  - $\boldsymbol{H}_c$ the design matrix,
  - $\Phi_c = \{\boldsymbol{\beta}_c, \boldsymbol{\psi}_c, \sigma_c^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_c the correlation function,
- notations for the **discrepancy**:
  - $\boldsymbol{H}_\delta$ the design matrix,
  - $\Phi_\delta = \{\boldsymbol{\beta}_\delta, \boldsymbol{\psi}_\delta, \sigma_\delta^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_δ the correlation function,
- $\Phi = \{\Phi_c, \Phi_\delta\}$,
- $\boldsymbol{y} = (\boldsymbol{y}_{exp}, \boldsymbol{y}_c)^T$ of size $T + N$.

Calibration

Likelihoods

Notations

- let us consider D a N sized DOE chosen in the space $\mathcal{H} \times \mathcal{Q}$ such that $\boldsymbol{y_c} = f_c(D)$,
- notations for the **surrogate**:
  - $\boldsymbol{H}_c$ the design matrix,
  - $\Phi_c = \{\boldsymbol{\beta}_c, \boldsymbol{\psi}_c, \sigma_c^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_c the correlation function,
- notations for the **discrepancy**:
  - $\boldsymbol{H}_\delta$ the design matrix,
  - $\Phi_\delta = \{\boldsymbol{\beta}_\delta, \boldsymbol{\psi}_\delta, \sigma_\delta^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_δ the correlation function,
- $\Phi = \{\Phi_c, \Phi_\delta\}$,
- $\boldsymbol{y} = (\boldsymbol{y}_{exp}, \boldsymbol{y}_c)^T$ of size $T + N$.

Calibration

Likelihoods

Notations

- let us consider D a N sized DOE chosen in the space $\mathcal{H} \times \mathcal{Q}$ such that $\boldsymbol{y_c} = f_c(D)$,
- notations for the **surrogate**:
  - $\boldsymbol{H}_c$ the design matrix,
  - $\Phi_c = \{\boldsymbol{\beta}_c, \boldsymbol{\psi}_c, \sigma_c^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_c the correlation function,
- notations for the **discrepancy**:
  - $\boldsymbol{H}_\delta$ the design matrix,
  - $\Phi_\delta = \{\boldsymbol{\beta}_\delta, \boldsymbol{\psi}_\delta, \sigma_\delta^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_δ the correlation function,
- $\Phi = \{\Phi_c, \Phi_\delta\}$,
- $\boldsymbol{y} = (\boldsymbol{y}_{exp}, \boldsymbol{y}_c)^T$ of size $T + N$.

Calibration

Likelihoods

Notations

- let us consider D a N sized DOE chosen in the space $\mathcal{H} \times \mathcal{Q}$ such that $\boldsymbol{y_c} = f_c(D)$,
- notations for the **surrogate**:
  - $\boldsymbol{H}_c$ the design matrix,
  - $\Phi_c = \{\boldsymbol{\beta}_c, \boldsymbol{\psi}_c, \sigma_c^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_c the correlation function,
- notations for the **discrepancy**:
  - $\boldsymbol{H}_\delta$ the design matrix,
  - $\Phi_\delta = \{\boldsymbol{\beta}_\delta, \boldsymbol{\psi}_\delta, \sigma_\delta^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_δ the correlation function,
- $\Phi = \{\Phi_c, \Phi_\delta\}$,
- $\boldsymbol{y} = (\boldsymbol{y}_{exp}, \boldsymbol{y}_c)^T$ of size $T + N$.

Calibration

Likelihoods

Notations

- let us consider D a N sized DOE chosen in the space $\mathcal{H} \times \mathcal{Q}$ such that $\boldsymbol{y_c} = f_c(D)$,
- notations for the **surrogate**:
  - $\boldsymbol{H}_c$ the design matrix,
  - $\Phi_c = \{\boldsymbol{\beta}_c, \boldsymbol{\psi}_c, \sigma_c^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_c the correlation function,
- notations for the **discrepancy**:
  - $\boldsymbol{H}_\delta$ the design matrix,
  - $\Phi_\delta = \{\boldsymbol{\beta}_\delta, \boldsymbol{\psi}_\delta, \sigma_\delta^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_δ the correlation function,
- $\Phi = \{\Phi_c, \Phi_\delta\}$,
- $\boldsymbol{y} = (\boldsymbol{y}_{exp}, \boldsymbol{y}_c)^T$ of size $T + N$.

Calibration

Likelihoods

Notations

- let us consider D a N sized DOE chosen in the space $\mathcal{H} \times \mathcal{Q}$ such that $\boldsymbol{y_c} = f_c(D)$,
- notations for the **surrogate**:
  - $\boldsymbol{H}_c$ the design matrix,
  - $\Phi_c = \{\boldsymbol{\beta}_c, \boldsymbol{\psi}_c, \sigma_c^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_c the correlation function,
- notations for the **discrepancy**:
  - $\boldsymbol{H}_\delta$ the design matrix,
  - $\Phi_\delta = \{\boldsymbol{\beta}_\delta, \boldsymbol{\psi}_\delta, \sigma_\delta^2\}$ the coefficient vector, the regularity of the kernel and the variance,
  - r_δ the correlation function,
- $\Phi = \{\Phi_c, \Phi_\delta\}$,
- $\boldsymbol{y} = (\boldsymbol{y}_{exp}, \boldsymbol{y}_c)^T$ of size $T + N$.

Calibration

Full likelihood

Not time consuming code $\rightarrow$ the full likelihood

$$\boldsymbol{y}_{exp}, \boldsymbol{X} | \boldsymbol{\theta}, \sigma^2_{err} \sim \mathcal{N}_{\color{red}{T}}(\boldsymbol{m}(\boldsymbol{X}, \boldsymbol{\theta}), \boldsymbol{V}(\boldsymbol{X})).$$

$$\mathcal{L}^F(\boldsymbol{\theta}; \boldsymbol{y}_{exp}, \boldsymbol{X}) = \frac{1}{(2\pi)^{T/2}|\boldsymbol{V}(\boldsymbol{X})|^{1/2}} \exp\left\{ -\frac{1}{2}\Big(\boldsymbol{y}_{exp} - \boldsymbol{m}(\boldsymbol{X}, \boldsymbol{\theta})\Big)^T \boldsymbol{V}(\boldsymbol{X})^{-1}\Big(\boldsymbol{y}_{exp} - \boldsymbol{m}(\boldsymbol{X}, \boldsymbol{\theta})\Big)\right\}.$$

Calibration

Full likelihood

Not time consuming code → the full likelihood

$$\boldsymbol{y}_{exp}, \boldsymbol{X}|\boldsymbol{\theta}, \sigma^2_{err} \sim \mathcal{N}_T(\boldsymbol{m}(\boldsymbol{X}, \boldsymbol{\theta}), \boldsymbol{V}(\boldsymbol{X})).$$

$$\mathcal{L}^F(\boldsymbol{\theta}; \boldsymbol{y}_{exp}, \boldsymbol{X}) = \frac{1}{(2\pi)^{T/2}|\boldsymbol{V}(\boldsymbol{X})|^{1/2}} \exp\left\{ -\frac{1}{2}\left(\boldsymbol{y}_{exp} - \boldsymbol{m}(\boldsymbol{X}, \boldsymbol{\theta})\right)^T \boldsymbol{V}(\boldsymbol{X})^{-1}\left(\boldsymbol{y}_{exp} - \boldsymbol{m}(\boldsymbol{X}, \boldsymbol{\theta})\right)\right\}.$$

$\mathcal{M}_1$	$\mathcal{M}_3$
$\boldsymbol{V}(X) = \sigma^2_{err} I_T$	$\boldsymbol{V}(X) = \sigma^2_{err} I_T + \boldsymbol{\Sigma}_\delta(\boldsymbol{X})$
$\boldsymbol{m}(\boldsymbol{X}, \boldsymbol{\theta}) = f_c(\boldsymbol{X}, \boldsymbol{\theta})$	$\boldsymbol{m}(\boldsymbol{X}, \boldsymbol{\theta}) = f_c(\boldsymbol{X}, \boldsymbol{\theta}) + \boldsymbol{H}_\delta(\boldsymbol{X})\beta_\delta$

Calibration
Full likelihood

Not time consuming code → the full likelihood

$$\boldsymbol{y}_{exp}, \boldsymbol{X} | \boldsymbol{\theta}, \sigma^2_{err}, \Phi_\delta \sim \mathcal{N}_T(\boldsymbol{m}(\boldsymbol{X}, \boldsymbol{\theta}), \boldsymbol{V}(\boldsymbol{X})).$$

$$\mathcal{L}^F(\boldsymbol{\theta}, \Phi_\delta; \boldsymbol{y}_{exp}, \boldsymbol{X}) = \frac{1}{(2\pi)^{T/2}|\boldsymbol{V}(\boldsymbol{X})|^{1/2}} \exp\left\{ -\frac{1}{2}\Big(\boldsymbol{y}_{exp} - \boldsymbol{m}(\boldsymbol{X}, \boldsymbol{\theta})\Big)^T \boldsymbol{V}(\boldsymbol{X})^{-1}\Big(\boldsymbol{y}_{exp} - \boldsymbol{m}(\boldsymbol{X}, \boldsymbol{\theta})\Big)\right\}.$$

$\mathcal{M}_1$	$\mathcal{M}_3$
$\boldsymbol{V}(X) = \sigma^2_{err} I_T$	$\boldsymbol{V}(X) = \sigma^2_{err} I_T + \boldsymbol{\Sigma}_\delta(\boldsymbol{X})$
$\boldsymbol{m}(\boldsymbol{X}, \boldsymbol{\theta}) = f_c(\boldsymbol{X}, \boldsymbol{\theta})$	$\boldsymbol{m}(\boldsymbol{X}, \boldsymbol{\theta}) = f_c(\boldsymbol{X}, \boldsymbol{\theta}) + \boldsymbol{H}_\delta(\boldsymbol{X})\boldsymbol{\beta}_\delta$

Calibration

Full likelihood

Time consuming code $\rightarrow$ the full likelihood

$$\boldsymbol{y}, \boldsymbol{X}|\boldsymbol{\theta}, \Phi, \sigma^2_{err} \sim \mathcal{N}_{T+N}(\boldsymbol{m}(\boldsymbol{X}, \boldsymbol{\theta}), \boldsymbol{V}(\boldsymbol{X}, \boldsymbol{\theta})).$$

Calibration

Full likelihood

Time consuming code → the full likelihood

$$\boldsymbol{y}, \boldsymbol{X}|\theta, \Phi, \sigma^2_{err} \sim \mathcal{N}_{T+N}(\boldsymbol{m}(\boldsymbol{X}, \theta), \boldsymbol{V}(\boldsymbol{X}, \theta)).$$

$\mathcal{M}_2$

$$\boldsymbol{V}(\boldsymbol{X}, \boldsymbol{\theta}) = \begin{pmatrix} \boldsymbol{\Sigma}_{exp,exp}(\boldsymbol{X}, \boldsymbol{\theta}) + \sigma^2_{err}\boldsymbol{I}_T & \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X}, \boldsymbol{\theta}), \boldsymbol{D}) \\ \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X}, \boldsymbol{\theta}), \boldsymbol{D})^T & \boldsymbol{\Sigma}_{c,c}(\boldsymbol{D}) \end{pmatrix}$$

$$\boldsymbol{m}(\boldsymbol{X}, \boldsymbol{\theta}) = \boldsymbol{H}(\boldsymbol{X}, \boldsymbol{\theta})\boldsymbol{\beta}_c = \begin{pmatrix} \boldsymbol{H}_c(\boldsymbol{X}, \boldsymbol{\theta}) \\ \boldsymbol{H}_c(D) \end{pmatrix} \boldsymbol{\beta}_c$$

$\mathcal{M}_4$

$$\boldsymbol{V}(\boldsymbol{X}, \boldsymbol{\theta}) = \begin{pmatrix} \boldsymbol{\Sigma}_{exp,exp}(\boldsymbol{X}, \boldsymbol{\theta}) + \sigma^2_{err}\boldsymbol{I}_T + \boldsymbol{\Sigma}_{\delta}(\boldsymbol{X}) & \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X}, \boldsymbol{\theta}), \boldsymbol{D}) \\ \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X}, \boldsymbol{\theta}), \boldsymbol{D})^T & \boldsymbol{\Sigma}_{c,c}(\boldsymbol{D}) \end{pmatrix}$$

$$\boldsymbol{m}(\boldsymbol{X}, \boldsymbol{\theta}) = \boldsymbol{H}(\boldsymbol{X}, \boldsymbol{\theta})\boldsymbol{\beta}_c = \begin{pmatrix} \boldsymbol{H}_c(\boldsymbol{X}, \boldsymbol{\theta}) & \boldsymbol{H}_{\delta}(\boldsymbol{X}) \\ \boldsymbol{H}_c(D) & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{\beta}_c \\ \boldsymbol{\beta}_{\delta} \end{pmatrix}$$

Calibration

Full likelihood

Time consuming code → the full likelihood

$$\boldsymbol{y}, \boldsymbol{X}|\theta, \Phi, \sigma^2_{err} \sim \mathcal{N}_{T+N}(\boldsymbol{m}(\boldsymbol{X},\theta), \boldsymbol{V}(\boldsymbol{X},\theta)).$$

$\mathcal{M}_2$

$$\boldsymbol{V}(\boldsymbol{X},\boldsymbol{\theta}) = \begin{pmatrix} \boldsymbol{\Sigma}_{exp,exp}(\boldsymbol{X},\boldsymbol{\theta}) + \sigma^2_{err}\boldsymbol{I}_T & \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D}) \\ \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})^T & \boldsymbol{\Sigma}_{c,c}(\boldsymbol{D}) \end{pmatrix}$$

$$\boldsymbol{m}(\boldsymbol{X},\boldsymbol{\theta}) = \boldsymbol{H}(\boldsymbol{X},\boldsymbol{\theta})\boldsymbol{\beta}_c = \begin{pmatrix} \boldsymbol{H}_c(\boldsymbol{X},\boldsymbol{\theta}) \\ \boldsymbol{H}_c(D) \end{pmatrix} \boldsymbol{\beta}_c$$

$\mathcal{M}_4$

$$\boldsymbol{V}(\boldsymbol{X},\boldsymbol{\theta}) = \begin{pmatrix} \boldsymbol{\Sigma}_{exp,exp}(\boldsymbol{X},\boldsymbol{\theta}) + \sigma^2_{err}\boldsymbol{I}_T + \boldsymbol{\Sigma}_{\delta}(\boldsymbol{X}) & \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D}) \\ \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})^T & \boldsymbol{\Sigma}_{c,c}(\boldsymbol{D}) \end{pmatrix}$$

$$\boldsymbol{m}(\boldsymbol{X},\boldsymbol{\theta}) = \boldsymbol{H}(\boldsymbol{X},\boldsymbol{\theta})\boldsymbol{\beta}_c = \begin{pmatrix} \boldsymbol{H}_c(\boldsymbol{X},\boldsymbol{\theta}) & \boldsymbol{H}_{\delta}(\boldsymbol{X}) \\ \boldsymbol{H}_c(D) & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{\beta}_c \\ \boldsymbol{\beta}_{\delta} \end{pmatrix}$$

Calibration

Partial likelihood

Time consuming code → the partial likelihood

$$\boldsymbol{y}_c|\Phi_c \sim \mathcal{N}_N(\boldsymbol{m}_c, \boldsymbol{V}_c).$$

Calibration

Partial likelihood

Time consuming code → the partial likelihood

$$\boldsymbol{y}_c | \Phi_c \sim \mathcal{N}_N(\boldsymbol{m}_c, \boldsymbol{V}_c).$$

$\mathcal{M}_2$ $\mathcal{M}_4$

$$\boldsymbol{V}_c = \boldsymbol{\Sigma}_{c,c}(\boldsymbol{D})$$

$$\boldsymbol{m}_c = \boldsymbol{H}_c(\boldsymbol{D})\boldsymbol{\beta}_c$$

Calibration

Conditional likelihood

Time consuming code $\rightarrow$ the conditional distribution $[\boldsymbol{y}_{exp}|\boldsymbol{y}_c]$

$$\begin{pmatrix}\boldsymbol{y}_{exp}\\ \boldsymbol{y}_c\end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix}\boldsymbol{m}_{exp}(\boldsymbol{X},\boldsymbol{\theta})\\ \boldsymbol{m}_c\end{pmatrix}, \begin{pmatrix}\boldsymbol{\Sigma}_{exp,exp}(\boldsymbol{X},\boldsymbol{\theta}) & \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})\\ \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})^T & \boldsymbol{\Sigma}_{c,c}(\boldsymbol{D})\end{pmatrix}\right)$$

Calibration

Conditional likelihood

Time consuming code $\rightarrow$ the conditional distribution $[\boldsymbol{y}_{exp}|\boldsymbol{y}_c]$

$$\begin{pmatrix}\boldsymbol{y}_{exp}\\ \boldsymbol{y}_c\end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix}\boldsymbol{m}_{exp}(\boldsymbol{X},\boldsymbol{\theta})\\ \boldsymbol{m}_c\end{pmatrix}, \begin{pmatrix}\boldsymbol{\Sigma}_{exp,exp}(\boldsymbol{X},\boldsymbol{\theta}) & \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})\\ \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})^T & \boldsymbol{\Sigma}_{c,c}(\boldsymbol{D})\end{pmatrix}\right)$$

Then,

$$\boldsymbol{y}_{exp}|\boldsymbol{y}_c \sim \mathcal{N}(\boldsymbol{\mu}_{exp|c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D}), \boldsymbol{\Sigma}_{exp|c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})),$$

Calibration

Conditional likelihood

Time consuming code $\rightarrow$ the conditional distribution $[\boldsymbol{y}_{exp}|\boldsymbol{y}_c]$

$$\begin{pmatrix}\boldsymbol{y}_{exp}\\ \boldsymbol{y}_c\end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix}\boldsymbol{m}_{exp}(\boldsymbol{X},\boldsymbol{\theta})\\ \boldsymbol{m}_c\end{pmatrix}, \begin{pmatrix}\boldsymbol{\Sigma}_{exp,exp}(\boldsymbol{X},\boldsymbol{\theta}) & \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})\\ \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})^T & \boldsymbol{\Sigma}_{c,c}(\boldsymbol{D})\end{pmatrix}\right)$$

Then,

$$\boldsymbol{y}_{exp}|\boldsymbol{y}_c \sim \mathcal{N}(\boldsymbol{\mu}_{exp|c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D}),\boldsymbol{\Sigma}_{exp|c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})),$$

with:

$$\boldsymbol{\mu}_{exp|c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D}) = \boldsymbol{m}_{exp}(\boldsymbol{X},\boldsymbol{\theta}) + \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})\boldsymbol{\Sigma}_{c,c}(\boldsymbol{D})^{-1}(\boldsymbol{y}_c - \boldsymbol{m}_c(\boldsymbol{D})),$$

Calibration

Conditional likelihood

Time consuming code → the conditional distribution $[\boldsymbol{y}_{exp}|\boldsymbol{y}_c]$

$$\begin{pmatrix} \boldsymbol{y}_{exp} \\ \boldsymbol{y}_c \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \boldsymbol{m}_{exp}(\boldsymbol{X},\boldsymbol{\theta}) \\ \boldsymbol{m}_c \end{pmatrix}, \begin{pmatrix} \boldsymbol{\Sigma}_{exp,exp}(\boldsymbol{X},\boldsymbol{\theta}) & \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D}) \\ \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})^T & \boldsymbol{\Sigma}_{c,c}(\boldsymbol{D}) \end{pmatrix} \right)$$

Then,

$$\boldsymbol{y}_{exp}|\boldsymbol{y}_c \sim \mathcal{N}(\boldsymbol{\mu}_{exp|c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D}), \boldsymbol{\Sigma}_{exp|c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})),$$

with:

$$\boldsymbol{\mu}_{exp|c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D}) = \boldsymbol{m}_{exp}(\boldsymbol{X},\boldsymbol{\theta}) + \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})\boldsymbol{\Sigma}_{c,c}(\boldsymbol{D})^{-1}(\boldsymbol{y}_c - \boldsymbol{m}_c(\boldsymbol{D})),$$

$$\boldsymbol{\Sigma}_{exp|c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D}) = \boldsymbol{\Sigma}_{exp,exp}(\boldsymbol{X},\boldsymbol{\theta}) - \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})\boldsymbol{\Sigma}_{c,c}(\boldsymbol{D})^{-1}\boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})^T.$$

Calibration

Conditional likelihood

Time consuming code → the conditional distribution $[\boldsymbol{y}_{exp}|\boldsymbol{y}_c]$

$$\begin{pmatrix}\boldsymbol{y}_{exp}\\ \boldsymbol{y}_c\end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix}\boldsymbol{m}_{exp}(\boldsymbol{X},\boldsymbol{\theta})\\ \boldsymbol{m}_c\end{pmatrix}, \begin{pmatrix}\boldsymbol{\Sigma}_{exp,exp}(\boldsymbol{X},\boldsymbol{\theta}) & \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})\\ \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})^T & \boldsymbol{\Sigma}_{c,c}(\boldsymbol{D})\end{pmatrix}\right)$$

Then,

$$\boldsymbol{y}_{exp}|\boldsymbol{y}_c \sim \mathcal{N}(\boldsymbol{\mu}_{exp|c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D}), \boldsymbol{\Sigma}_{exp|c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})),$$

with:

$$\boldsymbol{\mu}_{exp|c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D}) = \boldsymbol{m}_{exp}(\boldsymbol{X},\boldsymbol{\theta}) + \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})\boldsymbol{\Sigma}_{c,c}(\boldsymbol{D})^{-1}(\boldsymbol{y}_c - \boldsymbol{m}_c(\boldsymbol{D})),$$

$$\boldsymbol{\Sigma}_{exp|c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D}) = \boldsymbol{\Sigma}_{exp,exp}(\boldsymbol{X},\boldsymbol{\theta}) - \boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})\boldsymbol{\Sigma}_{c,c}(\boldsymbol{D})^{-1}\boldsymbol{\Sigma}_{exp,c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})^T.$$

Conditional likelihood

$$\boldsymbol{y}_{exp}|\boldsymbol{y}_c, \boldsymbol{X}, \boldsymbol{D}, \boldsymbol{\theta}, \Phi, \sigma^2_{err} \sim \mathcal{N}_T(\boldsymbol{\mu}_{exp|c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D}), \boldsymbol{\Sigma}_{exp|c}((\boldsymbol{X},\boldsymbol{\theta}),\boldsymbol{D})).$$

Calibration

Different kind of estimations

- Maximum likelihood estimator (SMLE) (Cox et al., 2001; Wong et al., 2017),
- Bayesian framework (Bayarri et al., 2007; Kennedy and O'Hagan, 2001; Higdon et al., 2004).

Bayesian calibration

Estimation

main methods

- **joint estimation** (Higdon et al., 2004): all the parameters in Φ and σ^2_{err} are jointly estimated by maximizing the full likelihood $\mathcal{L}^F(\boldsymbol{\theta}, \Phi_\delta; \boldsymbol{y}, \boldsymbol{X})$ (after integration over β with a weak prior),

Bayesian calibration

Estimation

main methods

- **joint estimation** (Higdon et al., 2004): all the parameters in Φ and σ^2_{err} are jointly estimated by maximizing the full likelihood $\mathcal{L}^F(\boldsymbol{\theta}, \Phi_\delta; \boldsymbol{y}, \boldsymbol{X})$ (after integration over $\boldsymbol{\beta}$ with a weak prior),
- **modularization** (Liu et al., 2009) : Stepwise method
  - $(\hat{\Phi}_c) = \underset{\Phi_c}{argmax} \quad \mathcal{L}^P(\Phi_c|\boldsymbol{y}_c)$ (Maximum likelihood estimates),
  - $(\hat{\Phi}_c)$ are plugged into $\mathcal{L}^C(\boldsymbol{\theta}, \Phi_\delta; \Phi_c, \boldsymbol{y}_{exp}|\boldsymbol{y}_c, \boldsymbol{X})$,
  - $\mathcal{L}^C(\boldsymbol{\theta}, \Phi_\delta; \hat{\Phi}_c, \boldsymbol{y}_{exp}|\boldsymbol{y}_c, \boldsymbol{X})$ is used to sample the posterior density with MCMC methods.

Bayesian calibration

Estimation

main methods

- **joint estimation** (Higdon et al., 2004): all the parameters in Φ and σ^2_{err} are jointly estimated by maximizing the full likelihood $\mathcal{L}^F(\boldsymbol{\theta}, \Phi_{\delta}; \boldsymbol{y}, \boldsymbol{X})$ (after integration over β with a weak prior),
- **modularization** (Liu et al., 2009) : Stepwise method
  - $(\hat{\Phi}_c) = \underset{\Phi_c}{argmax} \quad \mathcal{L}^P(\Phi_c | \boldsymbol{y}_c)$ (Maximum likelihood estimates),
  - $(\hat{\Phi}_c)$ are plugged into $\mathcal{L}^C(\boldsymbol{\theta}, \Phi_{\delta}; \Phi_c, \boldsymbol{y}_{exp} | \boldsymbol{y}_c, \boldsymbol{X})$,
  - $\mathcal{L}^C(\boldsymbol{\theta}, \Phi_{\delta}; \hat{\Phi}_c, \boldsymbol{y}_{exp} | \boldsymbol{y}_c, \boldsymbol{X})$ is used to sample the posterior density with MCMC methods.

Bayesian calibration

Estimation

main methods

- **joint estimation** (Higdon et al., 2004): all the parameters in Φ and σ^2_{err} are jointly estimated by maximizing the full likelihood $\mathcal{L}^F(\boldsymbol{\theta}, \Phi_{\boldsymbol{\delta}}; \boldsymbol{y}, \boldsymbol{X})$ (after integration over β with a weak prior),
- **modularization** (Liu et al., 2009) : Stepwise method
  - $(\hat{\Phi}_c) = \underset{\Phi_c}{argmax} \quad \mathcal{L}^P(\Phi_c|\boldsymbol{y}_c)$ (Maximum likelihood estimates),
  - $(\hat{\Phi}_c)$ are plugged into $\mathcal{L}^C(\boldsymbol{\theta}, \Phi_{\boldsymbol{\delta}}; \Phi_c, \boldsymbol{y}_{exp}|\boldsymbol{y}_c, \boldsymbol{X})$,
  - $\mathcal{L}^C(\boldsymbol{\theta}, \Phi_{\delta}; \hat{\Phi}_c, \boldsymbol{y}_{exp}|\boldsymbol{y}_c, \boldsymbol{X})$ is used to sample the posterior density with MCMC methods.

Bayesian calibration

Estimation

main methods

- **joint estimation** (Higdon et al., 2004): all the parameters in Φ and σ_{err}^2 are jointly estimated by maximizing the full likelihood $\mathcal{L}^F(\boldsymbol{\theta}, \Phi_{\boldsymbol{\delta}}; \boldsymbol{y}, \boldsymbol{X})$ (after integration over $\boldsymbol{\beta}$ with a weak prior),
- **modularization** (Liu et al., 2009) : Stepwise method
  - $(\hat{\Phi}_c) = \underset{\Phi_c}{argmax} \quad \mathcal{L}^P(\Phi_c|\boldsymbol{y}_c)$ (Maximum likelihood estimates),
  - $(\hat{\Phi}_c)$ are plugged into $\mathcal{L}^C(\boldsymbol{\theta}, \Phi_{\boldsymbol{\delta}}; \Phi_c, \boldsymbol{y}_{exp}|\boldsymbol{y}_c, \boldsymbol{X})$,
  - $\mathcal{L}^C(\boldsymbol{\theta}, \Phi_{\boldsymbol{\delta}}; \hat{\Phi}_c, \boldsymbol{y}_{exp}|\boldsymbol{y}_c, \boldsymbol{X})$ is used to sample the posterior density with MCMC methods.

Context

I

Calibration in theory

- Mathematical definition of the numerical code
- Different statistical models
- Estimation methods

II

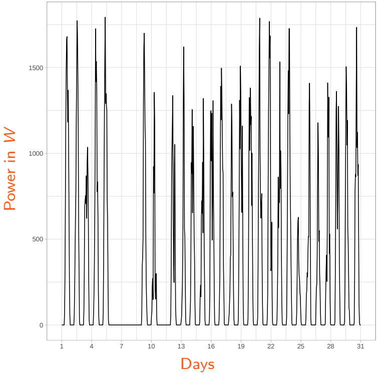
Application Case:
Code for the prediction of power from a PV plant (**Python code**)

Application case

Recorded data

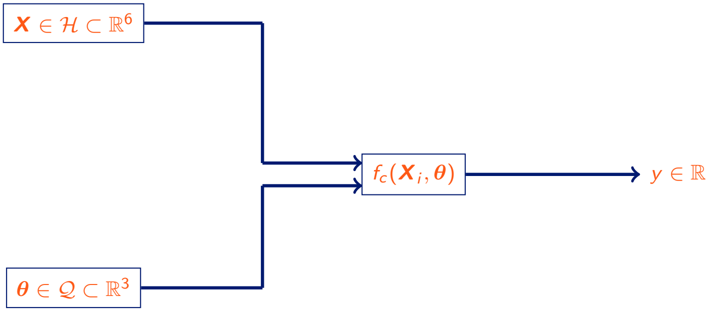
Application case

Recorded data



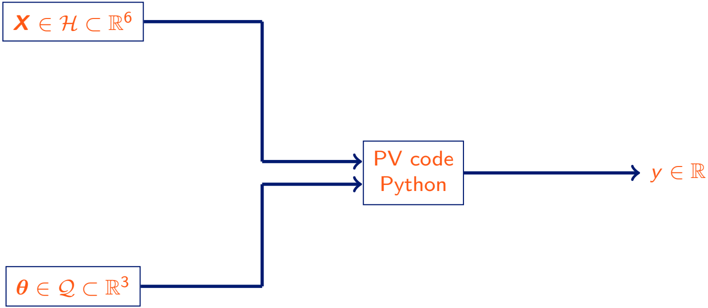
Application case

Presentation of the code



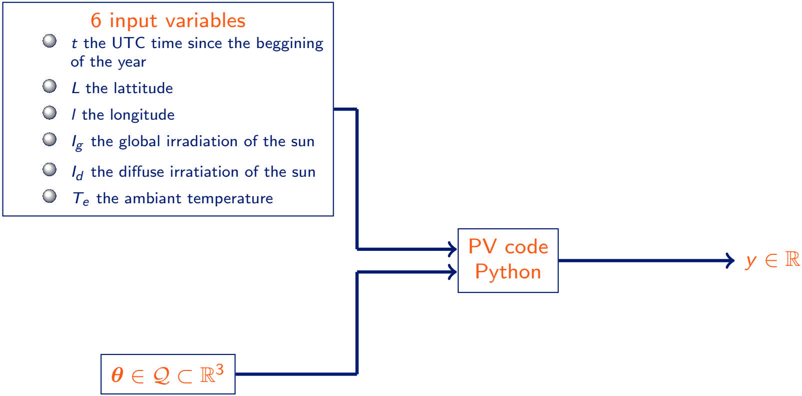
Application case

Presentation of the code



Application case

Presentation of the code



Application case

Presentation of the code

6 input variables

- t the UTC time since the beggining of the year
- L the lattitude
- l the longitude
- I_g the global irradiation of the sun
- I_d the diffuse irratiation of the sun
- T_e the ambiant temperature

3 parameters to calibrate

- η module photo-conversion efficiency
- μ_t module temperature coefficient (the efficiency decreases when the temperature rises)
- a_r describe the transmission of the radiation as a function of the incidence angle of solar rays, which depends on optical properties and the cleanliness

Python PV code → $y \in \mathbb{R}$

Application case

Presentation of the code

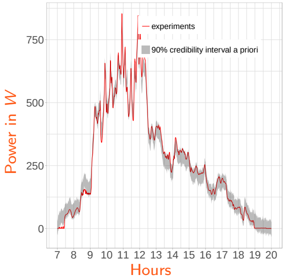
6 input variables

- t the UTC time since the beggining of the year
- L the lattitude
- l the longitude
- I_g the global irradiation of the sun
- I_d the diffuse irratiation of the sun
- T_e the ambiant temperature

3 parameters to calibrate

- η module photo-conversion efficiency
- μ_t module temperature coefficient (the efficiency decreases when the temperature rises)
- a_r describe the transmission of the radiation as a function of the incidence angle of solar rays, which depends on optical properties and the cleanliness

PV code Python



Application case

Python code

Modeling choices

- $m_\delta = 0$,
- r_δ as Gaussian,
- D is a LHS maximin of 50 points,
- $\boldsymbol{h}_c(\bullet, \bullet)$ composed of linear functions,
- r_c as Matérn 5/2,
- Prior densities :
  - $\eta \sim \mathcal{N}(0.143, 2.5.10^{-3})$,
  - $\mu_t \sim \mathcal{N}(-0.4, 1.10^{-2})$,
  - $a_r \sim \mathcal{N}(0.17, 3.6.10^{-3})$,
  - $\sigma^2_{err} \sim \Gamma(2, 169)$,
  - $\sigma^2_\delta \sim \Gamma(3, 1)$,
  - $\psi_\delta \sim \mathcal{U}(0, 1)$.

Application case

Python code

Modeling choices

- $m_\delta = 0$,
- r_δ as Gaussian,
- D is a LHS maximin of 50 points,
- $\boldsymbol{h}_c(\bullet, \bullet)$ composed of linear functions,
- r_c as Matérn 5/2,
- Prior densities :
  - $\eta \sim \mathcal{N}(0.143, 2.5.10^{-3})$,
  - $\mu_t \sim \mathcal{N}(-0.4, 1.10^{-2})$,
  - $a_r \sim \mathcal{N}(0.17, 3.6.10^{-3})$,
  - $\sigma^2_{err} \sim \Gamma(2, 169)$,
  - $\sigma^2_\delta \sim \Gamma(3, 1)$,
  - $\psi_\delta \sim \mathcal{U}(0, 1)$.

Application case

Python code

Modeling choices

- $m_\delta = 0$,
- r_δ as Gaussian,
- D is a LHS maximin of 50 points,
- $\boldsymbol{h}_c(\bullet, \bullet)$ composed of linear functions,
- r_c as Matérn 5/2,
- Prior densities :
  - $\eta \sim \mathcal{N}(0.143, 2.5.10^{-3})$,
  - $\mu_t \sim \mathcal{N}(-0.4, 1.10^{-2})$,
  - $a_r \sim \mathcal{N}(0.17, 3.6.10^{-3})$,
  - $\sigma^2_{err} \sim \Gamma(2, 169)$,
  - $\sigma^2_\delta \sim \Gamma(3, 1)$,
  - $\psi_\delta \sim \mathcal{U}(0, 1)$.

Application case

Python code

Modeling choices

- $m_\delta = 0$,
- r_δ as Gaussian,
- D is a LHS maximin of 50 points,
- $\boldsymbol{h}_c(\bullet, \bullet)$ composed of linear functions,
- r_c as Matérn 5/2,
- Prior densities :
  - $\eta \sim \mathcal{N}(0.143, 2.5.10^{-3})$,
  - $\mu_t \sim \mathcal{N}(-0.4, 1.10^{-2})$,
  - $a_r \sim \mathcal{N}(0.17, 3.6.10^{-3})$,
  - $\sigma^2_{err} \sim \Gamma(2, 169)$,
  - $\sigma^2_\delta \sim \Gamma(3, 1)$,
  - $\psi_\delta \sim \mathcal{U}(0, 1)$.

Application case

Python code

Modeling choices

- $m_\delta = 0$,
- r_δ as Gaussian,
- D is a LHS maximin of 50 points,
- $\boldsymbol{h}_c(\bullet, \bullet)$ composed of linear functions,
- r_c as Matérn 5/2,
- Prior densities :
  - $\eta \sim \mathcal{N}(0.143, 2.5.10^{-3})$,
  - $\mu_t \sim \mathcal{N}(-0.4, 1.10^{-2})$,
  - $a_r \sim \mathcal{N}(0.17, 3.6.10^{-3})$,
  - $\sigma^2_{err} \sim \Gamma(2, 169)$,
  - $\sigma^2_\delta \sim \Gamma(3, 1)$,
  - $\psi_\delta \sim \mathcal{U}(0, 1)$.

Application case

Python code

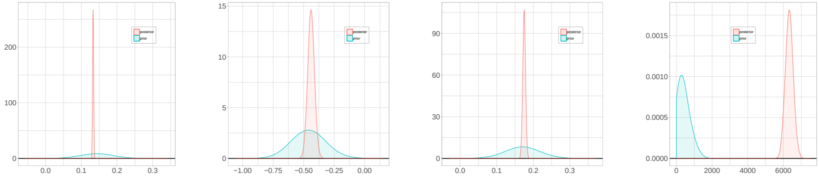
Modeling choices

- $m_\delta = 0$,
- r_δ as Gaussian,
- D is a LHS maximin of 50 points,
- $\boldsymbol{h}_c(\bullet, \bullet)$ composed of linear functions,
- r_c as Matérn 5/2,
- Prior densities :
  - $\eta \sim \mathcal{N}(0.143, 2.5.10^{-3})$,
  - $\mu_t \sim \mathcal{N}(-0.4, 1.10^{-2})$,
  - $a_r \sim \mathcal{N}(0.17, 3.6.10^{-3})$,
  - $\sigma^2_{err} \sim \Gamma(2, 169)$,
  - $\sigma^2_{\boldsymbol{\delta}} \sim \Gamma(3, 1)$,
  - $\psi_{\boldsymbol{\delta}} \sim \mathcal{U}(0, 1)$.

Bayesian calibration

Application case Results 1/3

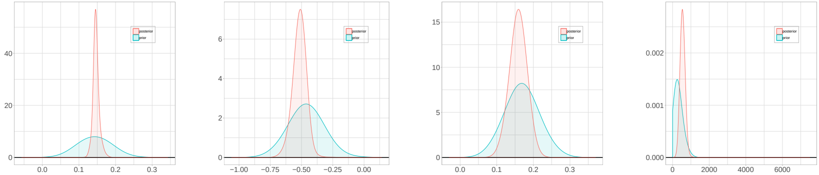
$\mathcal{M}_1$



η μ_t a_r σ^2_{err}

($\mathcal{M}_1$ to $\mathcal{M}_2$: Emulation of the code)

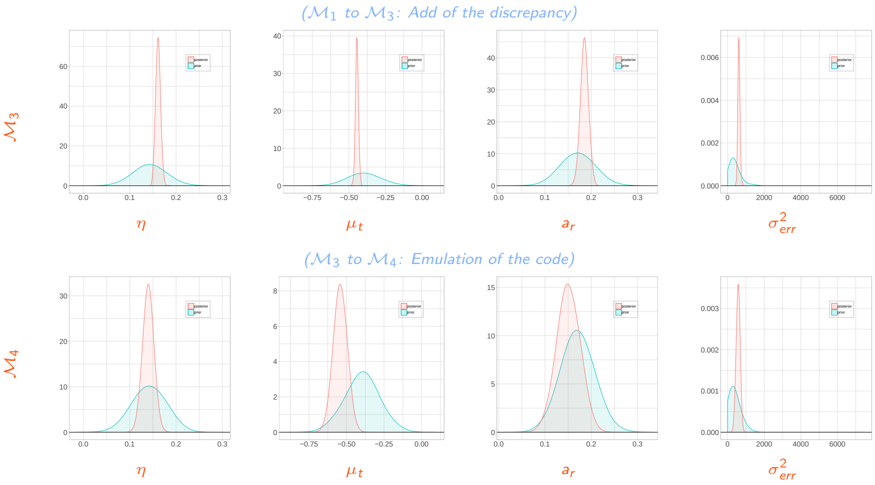
$\mathcal{M}_2$



η μ_t a_r σ^2_{err}

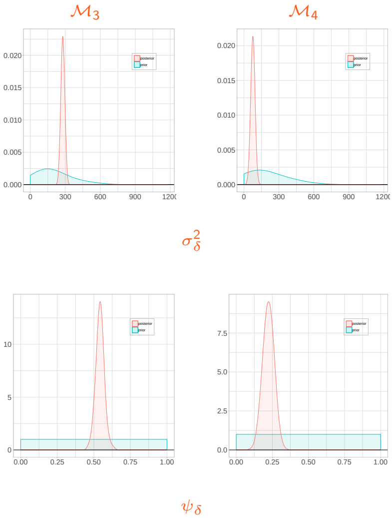
Bayesian calibration

Application case Results 2/3



Bayesian calibration

Application case Results 3/3



Bayesian calibration

Sequential design

Sequential design

- **Aim**: add selected points in the original design to improve calibration,
- some improvements can be performed on the GPE such as the Efficient Global Optimization algorithm (EGO (Jones et al., 1998)),
- Damblin et al. (2018) have used the principle of EGO to select points regarding further calibration.

Bayesian calibration

Sequential design

Sequential design

- **Aim**: add selected points in the original design to improve calibration,
- some improvements can be performed on the GPE such as the Efficient Global Optimization algorithm (EGO (Jones et al., 1998)),
- Damblin et al. (2018) have used the principle of EGO to select points regarding further calibration.

Bayesian calibration

Sequential design

Sequential design

- **Aim**: add selected points in the original design to improve calibration,
- some improvements can be performed on the GPE such as the Efficient Global Optimization algorithm (EGO (Jones et al., 1998)),
- Damblin et al. (2018) have used the principle of EGO to select points regarding further calibration.

Bayesian calibration

Sequential design

Sequential design

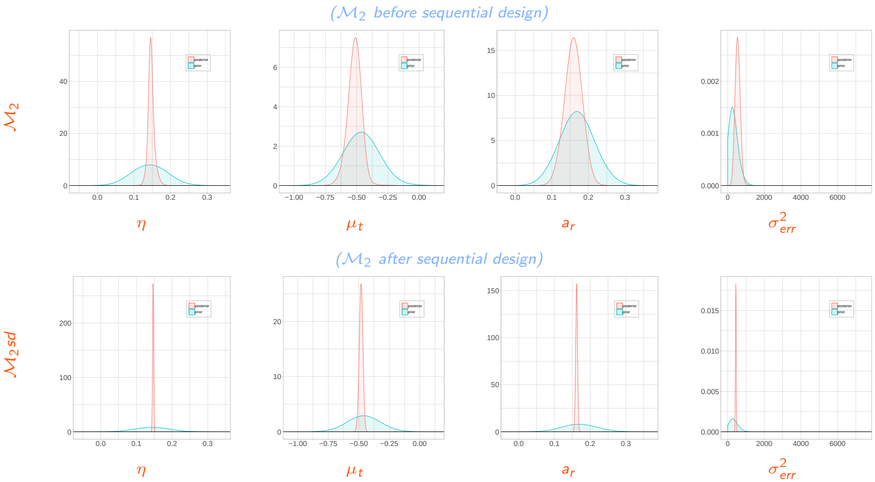
- **Aim**: add selected points in the original design to improve calibration,
- some improvements can be performed on the GPE such as the Efficient Global Optimization algorithm (EGO (Jones et al., 1998)),
- Damblin et al. (2018) have used the principle of EGO to select points regarding further calibration.

Improvement of the GPE

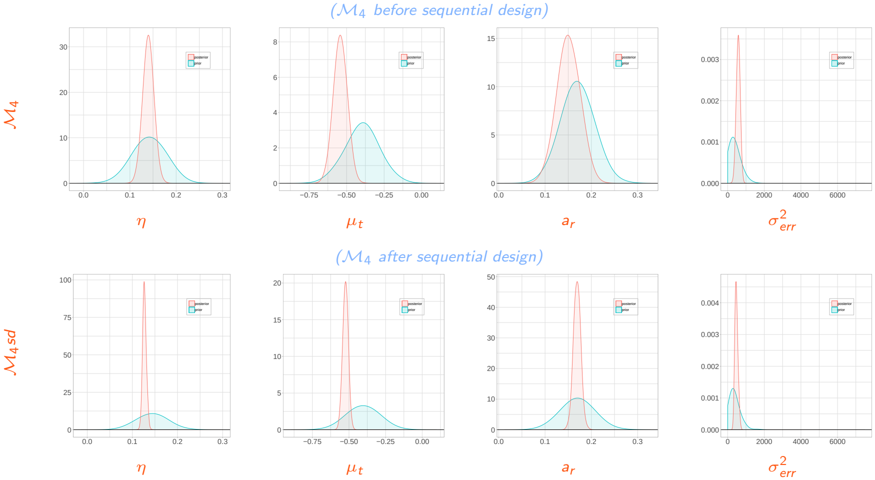
Choice: add 10 points in original design according to Damblin et al. (2018).

Bayesian calibration

Results with sequential design



Bayesian calibration
Results with sequential design



Conclusion

Conclusion

- Bayesian calibration better assessed the uncertainty on the parameters regarding experimental data,
- in the case of performance monitoring, Bayesian calibration works well,
- the article Carmassi et al. (2018), submitted available at arXiv:1801.01810, presents a review of main calibration methods,
- the package **CaliCo** (Carmassi, 2018) available on CRAN and at arXiv:1808.01932,
- a shiny application is built to help EDF employees apply Bayesian calibration on their different industrial cases.

Perspectives

Perspective

- Be placed in a case where the design of the project has to be done,
- use a real time consuming code on real sized PV plant,
- extend calibration on time series code outputs,
- use Bayesian Model Selection / Averaging to elect a better model, regarding the data (Damblin, 2015; Kamary, 2016).

References

Bayarri, M. J., Berger, J. O., Paulo, R., Sacks, J., Cafeo, J. A., Cavendish, J., Lin, C.-H., and Tu, J. (2007). A framework for validation of computer models. *Technometrics*, 49(2):138–154.

Carmassi, M. (2018). *CaliCo: Code Calibration in a Bayesian Framework*. R package version 0.1.0.

Carmassi, M., Barbillon, P., Chiodetti, M., Keller, M., and Parent, E. (2018). Bayesian calibration of a numerical code for prediction. *arXiv preprint arXiv:1801.01810*.

Cox, D. D., Park, J.-S., and Singer, C. E. (2001). A statistical method for tuning a computer code to a data base. *Computational statistics & data analysis*, 37(1):77–92.

Damblin, G. (2015). *Contributions statistiques au calage et à la validation des codes de calcul*. PhD thesis, PhD thesis, Université Paris Saclay.

Damblin, G., Barbillon, P., Keller, M., Pasanisi, A., and Parent, É. (2018). Adaptive numerical designs for the calibration of computer codes. *SIAM/ASA Journal on Uncertainty Quantification*, 6(1):151–179.

Giraud, C. (2014). *Introduction to high-dimensional statistics*. Chapman and Hall/CRC.

Higdon, D., Gattiker, J., Williams, B., and Rightley, M. (2008). Computer model calibration using high-dimensional output. *Journal of the American Statistical Association*, 103(482):570–583.

Higdon, D., Kennedy, M., Cavendish, J. C., Cafeo, J. A., and Ryne, R. D. (2004). Combining field data and computer simulations for calibration and prediction. *SIAM Journal on Scientific Computing*, 26(2):448–466.

Jones, D. R., Schonlau, M., and Welch, W. J. (1998). Efficient global optimization of expensive black-box functions. *Journal of Global optimization*, 13(4):455–492.

Kamary, K. (2016). *Non-informative priors and modelization by mixtures*. PhD thesis, PhD thesis, Université Paris Dauphine.

Kennedy, M. C. and O'Hagan, A. (2001). Bayesian calibration of computer models. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 63(3):425–464.

Liu, F., Bayarri, M., Berger, J., et al. (2009). Modularization in bayesian analysis, with emphasis on analysis of computer models. *Bayesian Analysis*, 4(1):119–150.

Sacks, J., Welch, W. J., Mitchell, T. J., and Wynn, H. P. (1989). Design and analysis of computer experiments. *Statistical science*, pages 409–423.

Wong, R. K., Storlie, C. B., and Lee, T. (2017). A frequentist approach to computer model calibration. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 79(2):635–648.