Informative Prior Distribution based on Experts Knowledge

Leave the objective bayes approaches alone

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Outline

An introductory example: Aircraft survivality

What are the vulnerable areas of the plane?[1]

Data

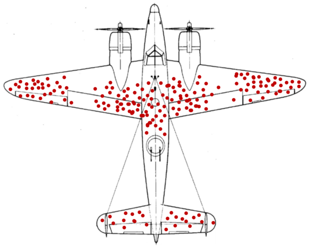
Planes returning with hits
Hits by types of ammunition

Model

Multinomial model with latent levels

Assumptions

- Equi-vulnerability areas
- "A hit will not down the plane" does not depend on the number of previous non-destructive hits



Main message: Survivorship bias

[1] Wald (1980) A Reprint of 'A Method of Estimating Plane Vulnerability Based on Damage of Survivors.' *(tech. report)*

HIV transmission

How to model the unsequenced diagnosed persons?[1]

Data

- Phylogenetic subtrees from:
  - Viral DNA sequences
  - Epidemiological data

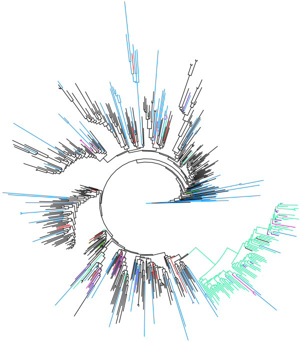
Subtree Size Models

- From branching process theory
- Bayesian nonparametric approach

Aims

- Track the epidemic spreading
- Determine risk factors related to introductions and transmissions

Main message: Important impact of data incompleteness



[1] PMG, Ratmann and Herbeck Manuscrit in preparation.

Truffle production

What is the impact of rainfall on truffle production?[1]

Data

Rainfall measures and truffle productions
for an orchard
from 1925 and to 1949

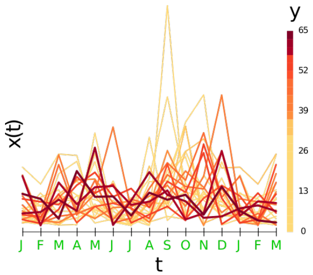
Model

Functional Linear Regression

Limitations

Difficulty to obtain data : only 25
Similar rainfall scenarios

Main message: need more data/information for a robust inference



[1] PMG, Abraham, Baragatti and Pudlo (2019) Bayesian Functional Linear Regression with Sparse Step Functions

Truffle production and modelling

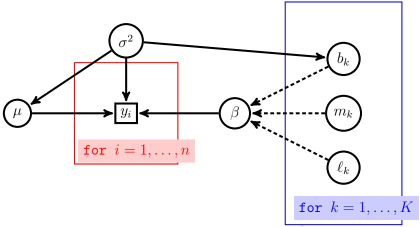
Functional Linear Regression model

$$y_i | x_i(\cdot), \mu, \beta(\cdot), \sigma^2 \overset{\text{ind}}{\sim} \mathcal{N}\left(\mu + \int_0^1 x_i(t)\beta(t)\mathrm{d}t, \sigma^2\right)$$

Parameter subspace: Stepfunctions

$$\beta(t) = \sum_{k=1}^{K} \frac{b_k}{|\mathcal{I}_k|} \mathbf{1}\{t \in \mathcal{I}_k\}$$

where $\mathcal{I}_k = [m_k + \ell_k; m_j - \ell_k]$



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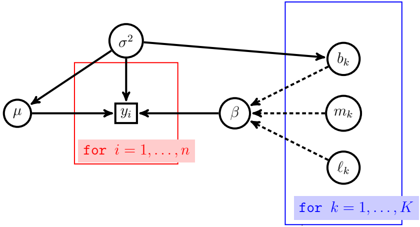

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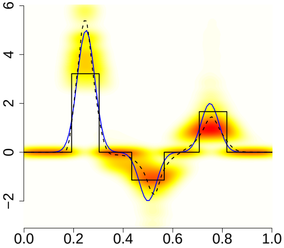
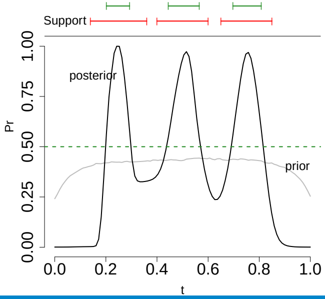
Estimates:

smooth function
stepfunction
support



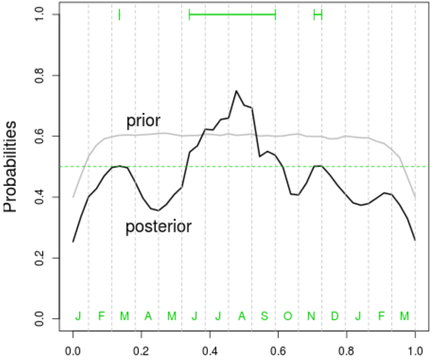


Application

Inference limitation:

prior probabilities ≈ posterior probabilities

too less data



Regression model and informative prior

Regression model: for $(y_i, x_i)_i$

$$y_i|x_i, \theta \sim P_\theta(\cdot|x_i)$$

Experts' knowledge:
Belief of expert e about θ is denoted by Π_e.
From a Bayesien point of view:

$$\pi_e(\theta) \propto p_\theta(\theta|\mathcal{D})\pi_0(\theta).$$

where $\mathcal{D}$ is the "expert experience".

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where $\mathcal{D}$ is the "expert experience".
Experts' predictive distribution: $p_e(\cdot | x)$

$$p_e(y|x) = \int p_\theta(y|x) \Pi_e(\mathrm{d}\theta | \mathcal{D})$$

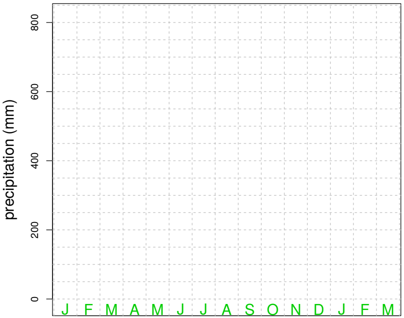
Linear model:
Linear Regression model is given by

$$y_i | x_i, \mu, \beta, \sigma^2 \overset{\text{ind}}{\sim} \mathcal{N}(\mu + \beta x_i, \sigma^2)$$

An expert's prediction can be viewed as:

$$\begin{aligned} \hat{y}_i^e &= \mathbb{E}_{\theta \sim \Pi_e(\cdot | \mathcal{D})}(y | x_i, \theta) \\ &= \mu_e + x_i \beta_e \end{aligned}$$

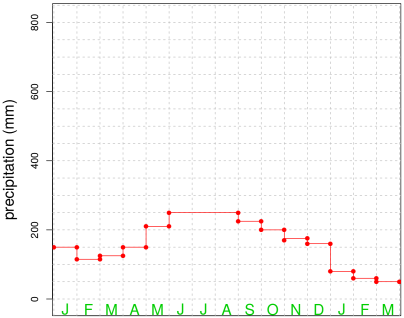
Give a rainfalls evolution $(x(t))$ and a likely production(y).

Rainfalls	Monthly rainfalls (mm)	Likely production (kilo per hectare)
	January :	
	February :	
	March :	
	April :	
	May :	
	June :	
	July :	
	August :	**Certainty**
	September :	
	October :	
	November :	
	Decembre :	
	January :	
	February :	
	March :	

Indirect elicitation

Give a rainfalls evolution $(x(t))$ and a likely production(y).

Rainfalls



Monthly rainfalls (mm)	Likely production (kilo per hectare)
January : **150**	
February : **115**	
March : **125**	
April : **150**	**70**
May : **210**	
June : **250**	
July : **250**	
August : **250**	**Certainty**
September : **225**	
October : **200**	
November : **175**	
Decembre : **160**	**0.8**
January : **80**	
February : **60**	
March : **50**	

Experts' uncertainty and combination

Pseudo-data:

Suppose that we have obtained predictions from E experts:

$(\hat{y}_i^e, x_i^e)$, for $e = 1, \ldots E$ and $i = 1, \ldots, n_e$

Prior distribution:

How to build an informative prior from these experts' predictions ?

- Uncertainty of the pseudo data
- Expert dependence structure
- Combination of experts' knowledge

Methods:

In the following, we present two approaches.

1- A hierarchical model[1]

2- A fractional approach[2,3]

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[1] Albert et al. (2012) Combining Expert Opinions in Prior Elicitation.

[2] Chen and Ibrahim (2000) Power prior distributions for regression models.

[3] Grünwald (2012) The Safe Bayesian.

Combination via a hierarchical prior

Experts' group:

Several consensus information

Pseudo data model:

$$y_i^{e,j} = \mu_{e,j} + \int_0^1 x_i^{e,j} \beta_{e,j} \mathrm{d}t$$

where $\varepsilon_i^{e,j} \overset{\text{ind}}{\sim} \mathcal{N}(0, \sigma_i^{e,j})$

Hierarchical prior:

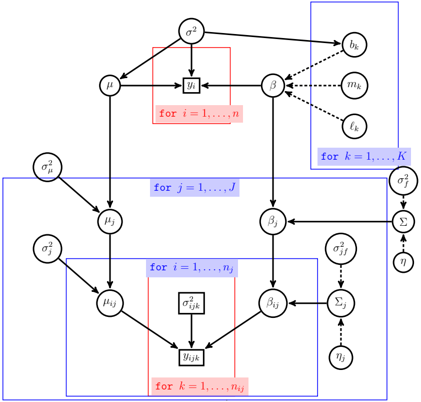
$$\mu_{e,j} \sim \mathcal{N}(\mu_j, \sigma_j) \qquad \beta_{e,j} \sim \mathcal{GP}(\beta_j, \Sigma_j)$$

$$\mu_j \sim \mathcal{N}(\mu, \sigma_\eta) \qquad \beta_j \sim \mathcal{GP}(\beta, \Sigma)$$

Covariance process:

$$\Sigma_j(u,t) = \sigma_{jf}^2 \exp\left\{ -\frac{1}{2\ell_j}(u-t)^2 \right\}$$

$$\Sigma(u,t) = \sigma_f^2 \exp\left\{ -\frac{1}{2\ell}(u-t)^2 \right\}$$



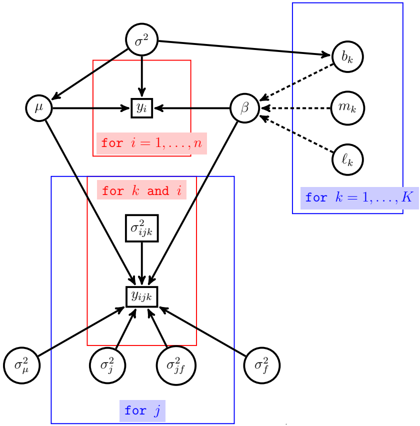
Combination via a hierarchical prior

Integrating out:

$\mu_{e,j}, \mu_j, \beta_{e,j}, \beta_j$

Unknown hyperparameters:

$\sigma_i^{e,j}, \sigma_j^2, \sigma_\eta^2, \sigma_{j,f}^2, \sigma_f^2, \ell_j, \ell$



Combination via a hierarchical prior

Integrating out:

$\mu_{e,j}, \mu_j, \beta_{e,j}, \beta_j$

Unknown hyperparameters:

$\sigma_i^{e,j}, \sigma_j^2, \sigma_\eta^2, \sigma_{j,f}^2, \sigma_f^2, \ell_j, \ell$

Noninformative prior:

$\pi(\sigma_f^2) \propto \sigma_f$, $\pi(\sigma_{j,f}^2) \propto \sigma_{j,f}$

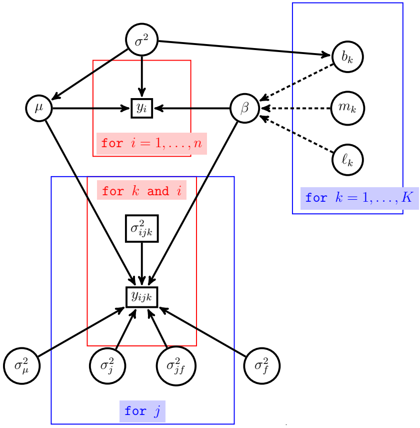
$\pi(\sigma_j^2) \propto \sigma_j$ and $\pi(\sigma_\eta^2) \propto \sigma_\eta$

Parameters tuning:

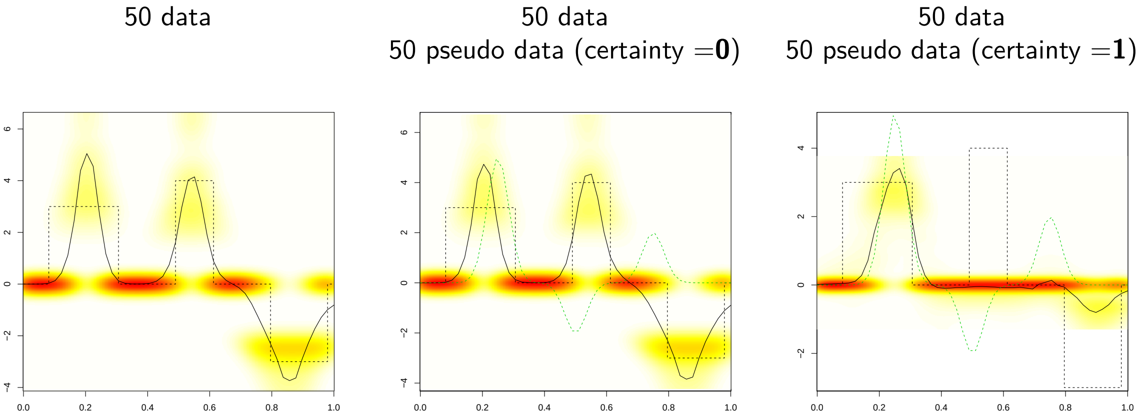
- ℓ_j and ℓ with additional information
- $\sigma_i^{e,j} = \dfrac{q^\star_{e,j}}{\Phi^{-1}\left(\frac{1+c_i^{e,j}}{2}\right)}$

where $c_i^{e,j}$ is the expert certainty and $q^\star_{e,j}$ is given by

$$P\left(\left|y_i^{e,j} - \mu_{e,j} - \int x_i^{e,j}(t)\beta_{e,j}(t)\mathrm{d}t\right| < q^\star_{e,j}\right) = c_i^{e,j}$$



Synthetic example



Fractional prior

Stepfunctions:

$$\beta(t) = \sum_{k=1}^{K} \frac{b_k}{|\mathcal{I}_k|} \mathbf{1}\{t \in \mathcal{I}_k\}$$

Bliss model: (data)

$$y_i | x_i(\cdot), \mu, b, \sigma^2, \mathcal{I} \overset{\text{ind}}{\sim} \mathcal{N}\left(\mu + x_i(\mathcal{I}_\cdot) b \;,\; \sigma^2\right)$$

Fractional prior

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$$\beta(t) = \sum_{k=1}^{K} \frac{b_k}{|\mathcal{I}_k|} \mathbf{1}\{t \in \mathcal{I}_k\}$$

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Pseudo-data model:

$$y_i^e | x_i^e(\cdot), \mu, b, \sigma^2, \mathcal{I} \overset{\text{ind}}{\sim} \mathcal{N}\left(\mu + x_i^e(\mathcal{I}_\cdot) b \; , \; \sigma^2\right)$$

Fractional prior: (given the pseudo data)

$$\pi_0(\theta) \times \prod_{e=1}^{E} \prod_{i=1}^{n_e} p(y_i^e | \theta)^{w_i^e} \propto \pi(\theta | y^1, \ldots, y^E; w)$$

Posterior properties (1/2)

Posterior

Posterior given observed data and pseudo data is proportional to

$$\left(\sigma^2\right)^{-\frac{1}{2}\left(n+\sum_{e=1}^{E} n_e w^e + K + 1\right)-1} \exp\left\{-\frac{1}{2\sigma^2}\left[\text{MSE} + \sum_{e=1}^{E} \text{MSE}_e + \mu^2 v_0^{-1} + b^T \Sigma(\mathcal{I})^{-1} b\right]\right\} \pi(\mathcal{I})$$

where

$$\text{MSE} = \sum_{i=1}^{n} \left(y_i - \mu - x_i(\mathcal{I}.)b\right)^2 \qquad \text{MSE}_e = \sum_{i=1}^{n_e} \boldsymbol{w_i^e} \left(y_i^e - \mu - x_i^e(\mathcal{I}.)b\right)^2$$

Posterior properties (1/2)

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Specific cases

Case 1 : **null weights**

$\Rightarrow$ posterior does **not depend on pseudo data**

Cas 2 : $\boldsymbol{w_i^e = 1}$

$\Rightarrow$ the pseudo datum **matters as an observed datum**.

Posterior properties (2/2)

$W^e = \mathrm{diag}\left(w_1^e, \ldots, w_{n_e}^e\right)$

Posterior expectation of b

$$\mathbb{E}\left(b|y, y^1, \ldots, y^E, \mathcal{I}\right) = M_w^{-1}\left[x(\mathcal{I})^T y + \sum_{e=1}^{E} x^e(\mathcal{I})^T W^e y^e\right]$$

where $M_w = \Sigma(\mathcal{I})^{-1} + x(\mathcal{I})^T x(\mathcal{I}) + \sum_{e=1}^{E} x^e(\mathcal{I})^T W^e x^e(\mathcal{I})$.

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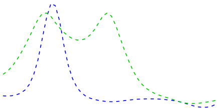
$\mathbb{E}\big(\beta(t)|y^1, \dots, y^E\big)$ - - -

$\mathbb{E}\big(\beta(t)|y\big)$ - - -



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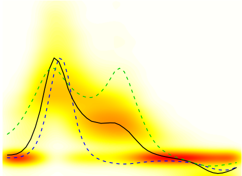
$\mathbb{E}\left(\beta(t) | y^1, \dots, y^E\right)$ - - -

$\mathbb{E}\left(\beta(t) | y\right)$ - - -

$\mathbb{E}\left(\beta(t) | y, y^1, \dots, y^E\right)$ ——



Posterior ——: Combination of

- **posterior given observed data** - - -
- **(fractional) posterior given pseudo data** - - -

Posterior properties (2/2)

$W^e = \text{diag}\left(w_1^e, \ldots, w_{n_e}^e\right)$

Posterior expectation of b

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$\mathbb{E}\left(\beta(t)|y^1, \ldots, y^E\right)$ - - -

$\mathbb{E}\left(\beta(t)|y\right)$ - - -

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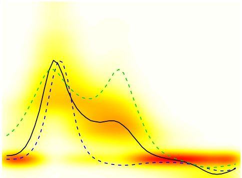

Weights decreasing

$\boldsymbol{w_i^e = 0.8}$

—— $\mathbb{E}\left(\beta(t)|y, y^1, \ldots, y^E\right)$

becomes

- - - $\mathbb{E}\left(\beta(t)|y\right)$

Posterior properties (2/2)

$W^e = \text{diag}\left(w_1^e, \ldots, w_{n_e}^e\right)$

Posterior expectation of b

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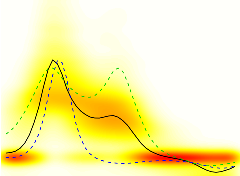
$\mathbb{E}\left(\beta(t)|y\right)$ - - -

$\mathbb{E}\left(\beta(t)|y, y^1, \ldots, y^E\right)$ ——

Weights decreasing

$\boldsymbol{w_i^e = 0.7}$



—— $\mathbb{E}\left(\beta(t)|y, y^1, \ldots, y^E\right)$

becomes

- - - $\mathbb{E}\left(\beta(t)|y\right)$

Posterior properties (2/2)

$W^e = \mathrm{diag}\left(w_1^e, \ldots, w_{n_e}^e\right)$

Posterior expectation of b

$$\mathbb{E}\left(b|y, y^1, \ldots, y^E, \mathcal{I}\right) = M_w^{-1}\left[x(\mathcal{I})^T y + \sum_{e=1}^{E} x^e(\mathcal{I})^T W^e y^e\right]$$

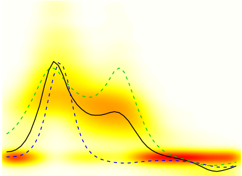
where $M_w = \Sigma(\mathcal{I})^{-1} + x(\mathcal{I})^T x(\mathcal{I}) + \sum_{e=1}^{E} x^e(\mathcal{I})^T W^e x^e(\mathcal{I})$.

$\mathbb{E}\left(\beta(t)|y^1, \ldots, y^E\right)$ - - -

$\mathbb{E}\left(\beta(t)|y\right)$ - - -

$\mathbb{E}\left(\beta(t)|y, y^1, \ldots, y^E\right)$ ——

Weights decreasing
$\boldsymbol{w_i^e = 0.6}$



—— $\mathbb{E}\left(\beta(t)|y, y^1, \ldots, y^E\right)$
becomes
- - - $\mathbb{E}\left(\beta(t)|y\right)$

Posterior properties (2/2)

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$\mathbb{E}\left(\beta(t)|y\right)$ - - -

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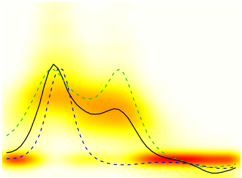

Weights decreasing

$\boldsymbol{w_i^e = 0.5}$

—— $\mathbb{E}\left(\beta(t)|y, y^1, \ldots, y^E\right)$

becomes

- - - $\mathbb{E}\left(\beta(t)|y\right)$

Posterior properties (2/2)

$W^e = \text{diag}\left(w_1^e, \ldots, w_{n_e}^e\right)$

Posterior expectation of b

$$\mathbb{E}\left(b|y, y^1, \ldots, y^E, \mathcal{I}\right) = M_w^{-1}\left[x(\mathcal{I})^T y + \sum_{e=1}^{E} x^e(\mathcal{I})^T W^e y^e\right]$$

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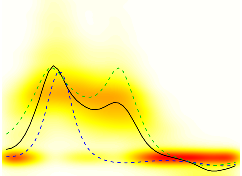

Weights decreasing

$\boldsymbol{w_i^e = 0.4}$

—— $\mathbb{E}\left(\beta(t)|y, y^1, \ldots, y^E\right)$

becomes

- - - $\mathbb{E}\left(\beta(t)|y\right)$

Posterior properties (2/2)

$W^e = \text{diag}\left(w_1^e, \ldots, w_{n_e}^e\right)$

Posterior expectation of b

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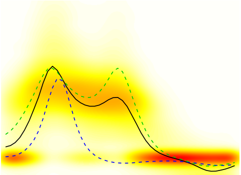
$\mathbb{E}\left(\beta(t)|y, y^1, \ldots, y^E\right)$ ——

Weights decreasing
$\boldsymbol{w_i^e = 0.3}$



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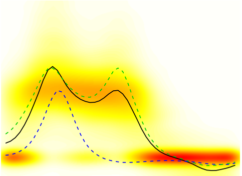
$\boldsymbol{w_i^e = 0.2}$



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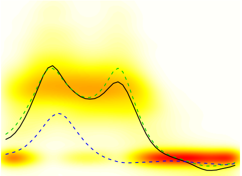

Weights decreasing

$\boldsymbol{w_i^e = 0.1}$

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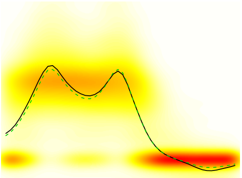

Weights decreasing

$\boldsymbol{w_i^e = 0}$

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The weight of the $i^{\mathbf{e}}$ pseudo datum of expert e : $w_i^e = c_i^e$?

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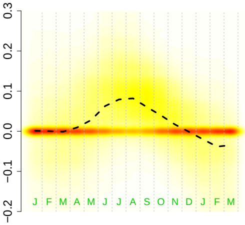
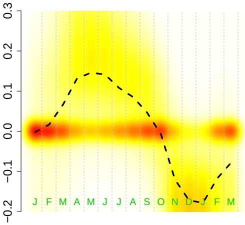
Application: truffle data

Prior of F. Le Tacon



Prior of J. Gravier



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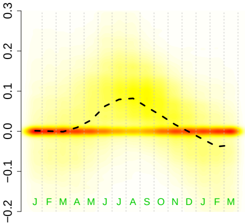

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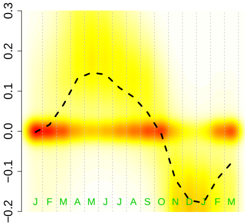
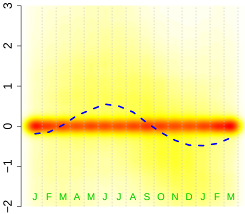


Prior of experts



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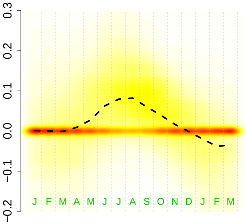

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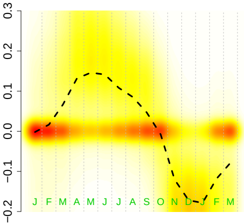

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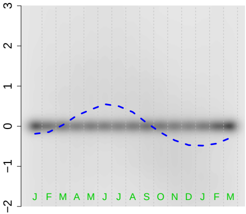

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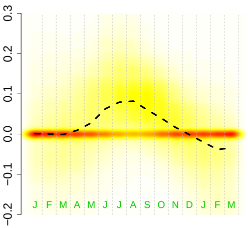

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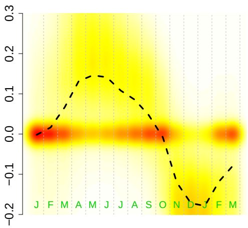

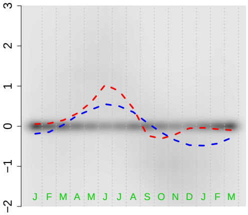

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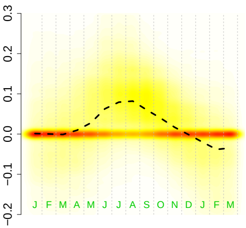

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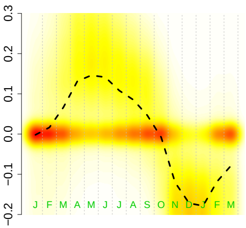
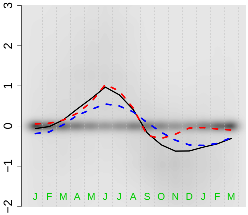


Posterior given pseudo data and observed data



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Future work

- Experts' dependence structure
- Improve the elicitation meetings
- Theoretical justification
- Design calibration

References

Grollemund, P. M., Abraham, C., Baragatti, M. and Pudlo, P. (2019) Bayesian Functional Linear Regression with Sparse Step Functions **(accepted)**

Grollemund, P. M., Abraham, C. and Baragatti, M. (2019+) Bayesian Functional Linear Regression with Informative Prior Distribution **(submitted)**

Le Tacon, F., Murat, C., Gravier, J., Montpied, P, Dupouey, J.-L., Grollemund, P.-M. and Baragatti, M (2018+) Evolution of the Périgord black truffle (Tuber melanosporum Vittad.) production in the Vaucluse department (France) from 1903 to 1988. Influence of annual climatic variations and possible effects of climate changes or sociological factors **(in revision)**

Implementation : CRAN: bliss package and github.com/pmgrollemund/bliss/